MAXIMUM LIKELIHOOD SEQUENCE ESTIMATION FOR MULTIPATH FADING CHANNELS

A Thesis

by

JOSEPH JAMES PAUTLER

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2000

Major Subject: Electrical Engineering

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ABSTRACT

Maximum Likelihood Sequence Estimation
for Multipath Fading Channels. (May 2000)

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This thesis addresses receiver design for multipath fading channels. The channel model was chosen to closely resemble a typical land mobile fading channel. Three receivers will be compared in this work. The first will be the optimum receiver in the maximum-likelihood (ML) sense. The optimum receiver represents the best possible bit error rate achievable. The second receiver will be based on the Expectation-Maximization (EM) algorithm. The EM algorithm is a recursive ML technique that can achieve close to optimal results with reduced complexity. The final receiver considered is the RAKE receiver. The RAKE receiver is a popular receiver structure that is currently being used for multipath fading channels. Simulation results are provided to compare the optimum and EM receivers and the RAKE and EM receivers. Simulations are also used to determine the performance of the EM receiver using different packet lengths and packet structures.

To my parents and Rachel

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I would first like to thank my advisor, Dr. Georghiades, for all of the time and advice that he has given me while I have been at Texas A&M. He has provided a solid basis for my studies. I would also like to thank Dr. Miller for our many discussions on wireless communications.

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CHAPTER I

INTRODUCTION

A. Problem Statement

This thesis addresses digital receiver design for multipath fading channels. The problem of communicating in multipath fading channels can be broken down into two separate issues. The first is that delayed versions of the transmitted signal are added together at the receiver to produce multipath interference. The second is that each major signal path is affected by a random amplitude fade and phase shift that tends to change over time. This work will consider the land mobile fading channel where the multipath interference and fading is caused by signals reflecting off buildings and other objects. The receiver is assumed to be moving around in this environment, as in a car driving down the road, so the fading is varying over time.

Three receivers will be considered in this work. The first will be the optimum receiver in the maximum-likelihood (ML) sense. The optimum receiver represents the best possible bit error rate achievable. The second receiver will be based on the Expectation-Maximization (EM) algorithm. The EM algorithm is a recursive ML technique that can achieve close to optimal results with reduced complexity. The final receiver considered is the RAKE receiver. The RAKE receiver is a popular receiver structure that is currently being used for multipath fading channels. The basic RAKE receiver assumes that the correlation between the transmitted signal and its delayed versions is negligible. The performance of the RAKE receiver is significantly degraded when compared with the optimum and EM based receivers if the signals are highly correlated.

The journal model is *IEEE Transactions on Automatic Control.*

B. Past Research

The issue of multipath interference was addressed by the work of Price and Green [1] who introduced the RAKE receiver structure. The RAKE receiver assumes that there is no correlation between the transmitted signal and its delayed versions. This type of multipath channel is known as a resolved multipath channel. Modern day CDMA systems that employ the RAKE receiver use wide bandwidth signals that have very low correlation when delayed. If the signals have significant correlation, the channel is considered unresolved and the RAKE receiver's performance will suffer. Methods for handling the unresolved case will be discussed later.

The issue of fading in a single path channel was addressed by Georghiades and Han [2]. The optimum receiver and an EM based receiver was derived and simulated for the Rayleigh fading channel. Receivers were derived for both slow and fast fading channels. In a slow fading channel, the fading variables stay constant over the entire data packet. In a fast fading channel, the fading variables change over the data packet with a certain degree of correlation. Fast Rayleigh fading represents the fading statistics that mobile receivers typically operate in. The mobile channel is also characterized by multipath. The work in [2] was not generalized to handle multipath channels.

The problem of multipath fading channels as a whole was addressed by Feder and Catipovic [3] and Danilo [4]. Feder and Catipovic investigated unresolved multipath Rayleigh fading in underwater channels and derived an EM based receiver for those channels. Danilo derived the optimum receiver for unresolved multipath Rayleigh and Rician fading channels. In each work, the fading was assumed to be static. This thesis will extend previous work to multipath non-static fading channels, which are representative of typical land mobile fading channels.

C. Objectives

- The optimum receiver in the maximum likelihood (ML) sense will be derived and simulated for an unresolved multipath non-static Rayleigh fading channel. A sub-optimum receiver based on the Expectation-Maximization (EM) algorithm will also be derived and simulated for the same channel. The first objective is to compare the error rate performance of the optimum and EM based receivers.

- The RAKE receiver is currently used in CDMA systems for resolved multipath channels. The RAKE receiver does not account for the correlation of the transmitted signal and its delayed versions. The second objective is to compare the error rate performance of the RAKE receiver to the EM based receiver in an unresolved multipath fading channel.

- The main advantage of the EM based receiver over the optimum receiver is the computational intensity. The EM based receiver can process much longer data packets than the optimum receiver. The third objective is to determine the effect of the data packet length and structure on the performance of the EM based receiver.

D. Contents of This Thesis

Chapter II gives in depth theoretical background on the land mobile fading channel, ML estimation, the EM algorithm, and the RAKE receiver. Chapter III contains the derivations of the optimum, EM, and RAKE receivers for the land mobile fading channel. Chapter IV describes the implementation of the channel and the receivers for the simulations. Chapter V provides simulation results that compare the receivers derived in Chapter III. Chapter VI is the conclusion. Appendix A includes a sample of the MATLAB code used for the EM based receiver simulations.

CHAPTER II

THEORETICAL BACKGROUND

This chapter provides an overview of the communications theory and estimation theory used in this thesis. A typical land mobile fading channel with multipath is presented in the first section. The next two sections introduce maximum-likelihood estimation and the Expectation-Maximization (EM) algorithm. An introduction to the RAKE receiver is presented in the last section.

A. Multipath Fading Channel

The multipath fading channel is one of the most difficult channels to communicate in. Multiple propagation paths converging at the receiver at significantly different times will cause signal distortion in the form of intersymbol interference (ISI). Each of the major propagation paths is also corrupted by random phase and amplitude fading. Complex receivers are necessary to accurately detect the bits [5–7].

Mobile receivers usually operate in multipath fading channels. This thesis focuses on a typical land mobile fading channel. Specifically, this thesis will consider a mobile station traveling along a road in a suburban neighborhood with no direct line of sight to the transmitting base station. Fig. 1 is a diagram of this scenario. Initially, the transmitted signal will bounce off large objects such as office buildings or hills. Before reaching the mobile, the signals will bounce off nearby houses and trees. In this model, there are two (or more) distinct signal paths that have significantly different arrival times. The difference of these arrival times is on the order of a symbol time. This form of multipath interference will cause ISI. The reflections from houses and trees surrounding the mobile will produce a random scattering of the incoming signals. Hundreds of scattered paths from each major path will arrive at the receiver at approximately the same time (much less than a symbol time). This form of multipath interference will cause the random phase and amplitude fading.

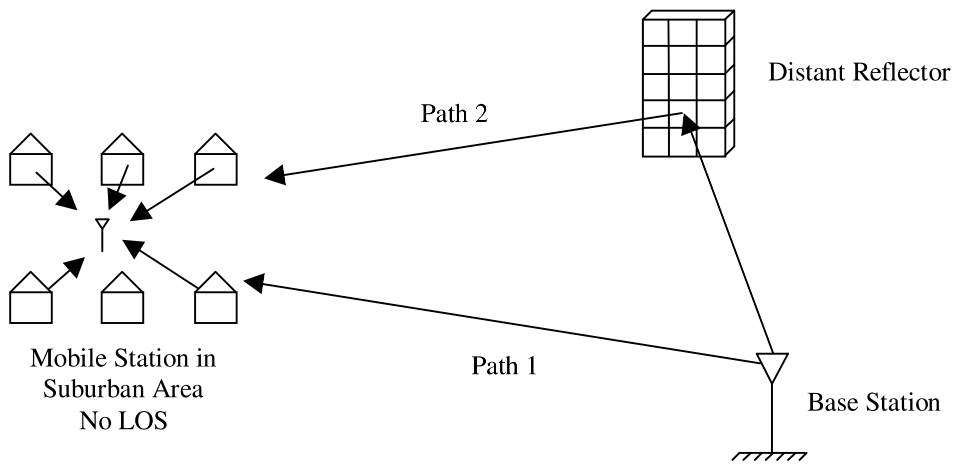

Fig. 1. Typical mobile environment.

The received signal in the given multipath fading model can be described mathematically as:

$$r(t) = \sum_{m=1}^{M} a_m(t)s(t-\tau_m) + n(t), \tag{2.1}$$

where $r(t)$ is the complex baseband received signal, $a_m(t)$ is the complex fading process for the mth path, $s(t)$ is the transmitted signal, τ_m is the path delay for the mth path, M is the total number of paths, and $n(t)$ is a white complex Gaussian process. The received signal model is also shown in Fig. 2.

The statistics of the multipath fading channel [8] are largely contained in the complex fading process, $a_m(t)$. From the given model, each significant signal path is scattered by the houses and trees near the mobile. Each leaf and branch of a tree and each brick and shingle on a house will create a different reflection. The process $a_m(t)$ is formed by adding thousands of reflected signals together that arrive at the receiver

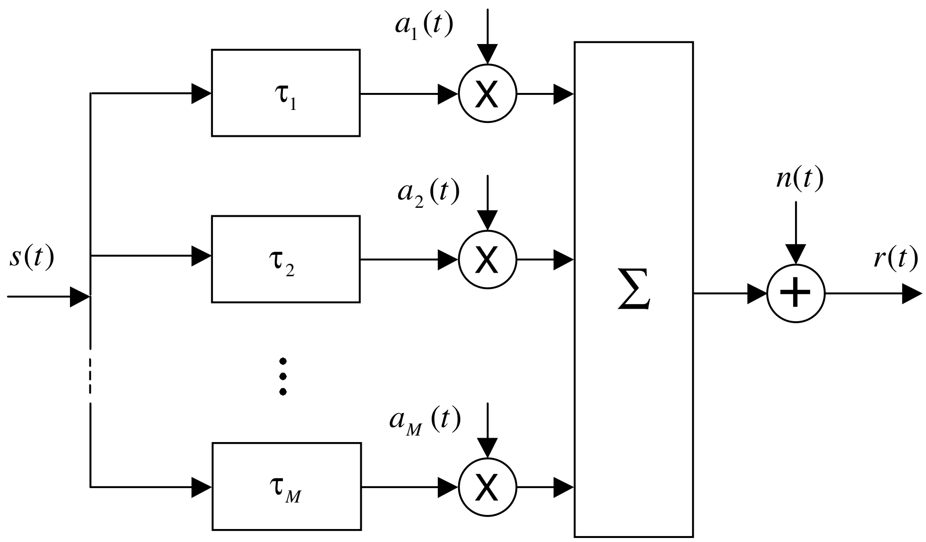

Fig. 2. Multipath signal structure.

with approximately the same path delay. Because there is a small time difference, the carrier phase of each reflection will be different. Therefore, some reflections will add constructively while others add destructively. The central limit theorem may be applied in this case. The result indicates that the process $a_m(t)$ will have a Gaussian distribution. The complex process $a_m(t)$ can be written in terms of its amplitude and phase:

$$a_m(t) = \rho_m(t)e^{j\theta_m(t)}. \tag{2.2}$$

It is well known that the amplitude of a complex Gaussian random variable has a Rayleigh distribution and the phase has a uniform distribution.

$$f_\rho(\rho) = \frac{\rho}{\sigma^2}\exp\left\{-\frac{\rho^2}{2\sigma^2}\right\}, \quad \rho \geq 0 \tag{2.3}$$

$$f_\Theta(\theta) = \frac{1}{2\pi}, \quad 0 < \theta \leq 2\pi \tag{2.4}$$

A fading process exhibiting this probability density function is known as *Rayleigh* fading.

The Rayleigh fading process $a_m(t)$ is also defined by its behavior over time. The change over time is caused by the mobile traveling through the environment. Fig. 3 is a diagram of a single reflected path arriving at the moving receiver.

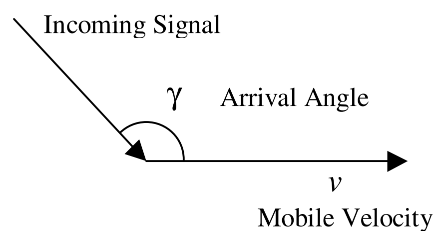

Fig. 3. Moving mobile and incident signal (top view).

The arriving signal will undergo a Doppler shift due to the motion of the receiver. The Doppler shift will be:

$$f = f_d \cos(\gamma). \tag{2.5}$$

The value f_d is the maximum Doppler shift defined as:

$$f_d = \frac{v}{c} f_c, \tag{2.6}$$

where v is the speed of the mobile, c is the speed of light, and f_c is the frequency of the transmitted signal.

Because a mobile in a suburban environment is surrounded by reflectors, there are incident signals from every direction. Because the angle of arrival is uniformly distributed, the power spectral density (PSD) for the fading process is:

$$S_a(f) = \frac{1}{\pi\sqrt{f_d^2 - f^2}} \qquad |f| < f_d. \tag{2.7}$$

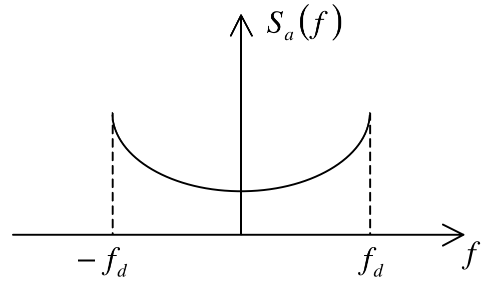

Fig. 4. PSD for the land mobile fading channel.

Fig. 4 shows the PSD of the complex fading process $a_m(t)$. The corresponding autocorrelation function is:

$$E\left[a_m(t)a_m^*(t-\tau)\right] = J_0(2\pi f_d \tau), \tag{2.8}$$

where $J_0(x)$ is the zeroth order Bessel function and $*$ represents the complex conjugate operation.

An example of a Rayleigh fading envelope is provided in Fig. 5. The Doppler shift for this plot is 100 Hz. The Doppler shift determines the rate at which the fading changes. Notice how the magnitude periodically drops down to a very low level. This is called a deep fade. The effects of a deep fade can be devastating to a receiver. The multipath channel can help combat the fading because more than one independently fading signal is available to the receiver. The probability that every path is in a deep fade is much less than the probability that one path is in a deep fade. This is a form of *diversity* [9].

B. Maximum-Likelihood Estimation

A maximum-likelihood (ML) parameter estimate [10,11] represents the most probable value a parameter $\mathbf{d}$ can take given that a certain observation $\mathbf{y}$ has been made. The

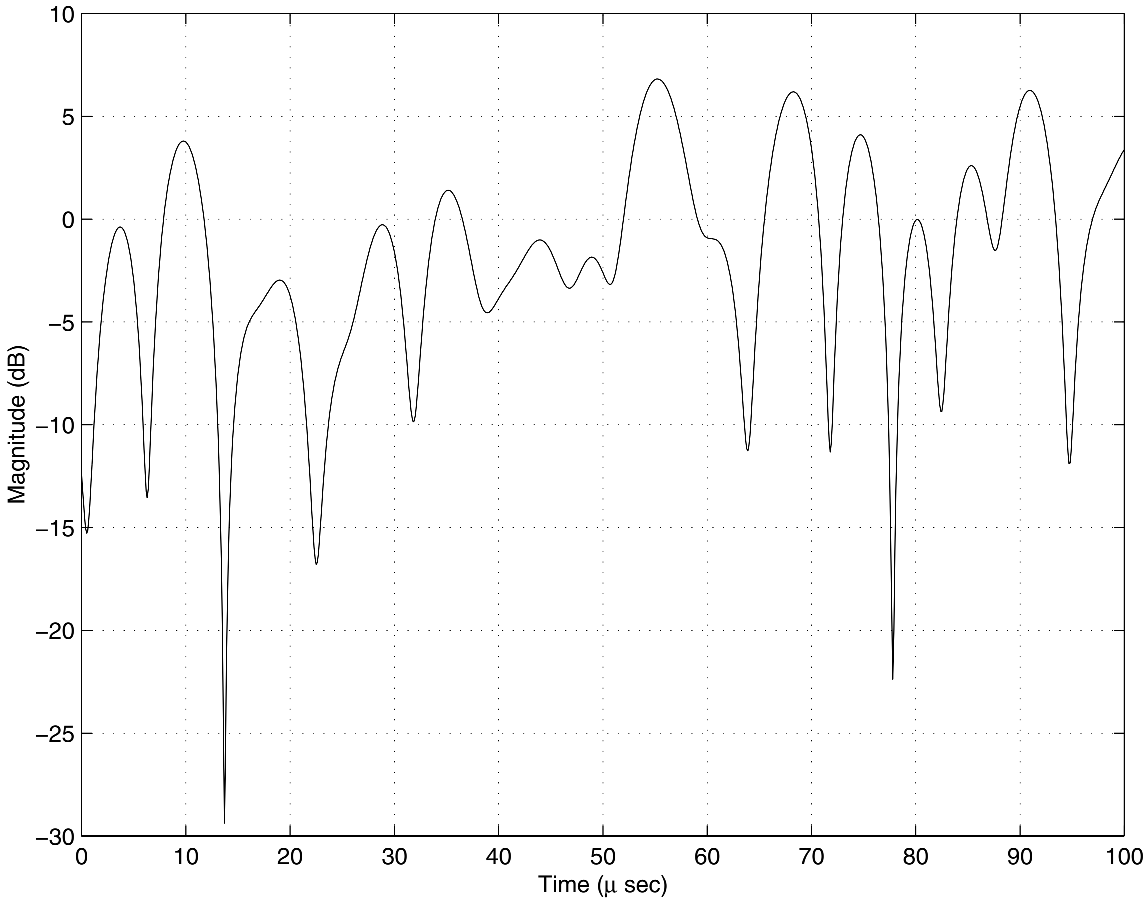

Fig. 5. Rayleigh fading envelope ($f_d = 100$ Hz).

process can be expressed mathematically as:

$$\hat{\mathbf{d}} = \arg \max_{\mathbf{d}} p(\mathbf{y}|\mathbf{d}), \tag{2.9}$$

where $p(\mathbf{y}|\mathbf{d})$ is the conditional density of $\mathbf{y}$ given $\mathbf{d}$, also known as the "likelihood" function. The ML estimate $\hat{\mathbf{d}}$ is the value of $\mathbf{d}$ that maximizes the likelihood function.

There are several different criteria used to define "optimum." One such criterion is the minimum mean square error (MMSE). It is a measure of "closeness." MMSE is useful for comparing two waveforms to each other. The criterion chosen depends on the application. For bit detection, the most useful criterion is the minimum probability of bit error. This is the maximum-likelihood bit estimate. For most digital communications applications, the optimum receiver refers to the receiver with the lowest probability of bit error. Such a receiver is optimum in the ML sense. Optimum receivers are often studied in the literature [2, 4, 12–15].

In many communications problems, the observed data will contain other random parameters, $\mathbf{a}$ (excluding the additive noise). The ML receiver must average out the effects of these parameters on the likelihood function. The ML receiver becomes:

$$\hat{\mathbf{d}} = \arg\max_{\mathbf{d}} E_{\mathbf{a}}\left[p(\mathbf{y}|\mathbf{d}, \mathbf{a})\right]. \tag{2.10}$$

There are some serious drawbacks to the ML receiver. The likelihood function will not always take on a closed form and its maximization usually involves an exhaustive search. For multipath fading channels, the likelihood function can be expressed in a closed form, but the maximization requires 2^N separate computations of the likelihood function where N is the number of bits in a packet. The exponential complexity renders the ML receiver hopelessly impractical for anything but extremely small packet lengths. A more practical alternative to the strict ML receiver is the Expectation-Maximization (EM) algorithm described in the next section.

C. EM Algorithm

The Expectation-Maximization (EM) algorithm is an iterative ML procedure that is well defined in the statistics literature by [16, 17]. Applications of the EM algorithm can be found in [2,3,18–23]. The benefits of this algorithm are reduced computational complexity and the ability to converge to the true ML estimate. The drawback is that the algorithm sometimes converges to a local maximum instead of the global maximum. A description of the general EM algorithm is given below.

The problem setup is this: a set of observed data, $\mathbf{y}$, is used to estimate a set of parameters, $\mathbf{d}$. The conditional density of $\mathbf{y}$ given $\mathbf{d}$ can be very difficult (or impossible) to write in closed form. If it is possible to obtain a different set of observations, $\mathbf{x}$, then it might be much easier to write the conditional density of $\mathbf{x}$ given $\mathbf{d}$. The make-believe set of data $\mathbf{x}$ is known as the *complete* data while the available observed data $\mathbf{y}$ is known as the *incomplete* data. The complete data $\mathbf{x}$ must be chosen so a many-to-one mapping $\mathbf{x} \rightarrow \mathbf{y}$ is made. The EM algorithm is a two-step procedure that uses the conditional density of the complete data instead of the incomplete data. The ith iteration of the algorithm can be summarized in these two steps:

1. Expectation step: Compute $Q(\mathbf{d}|\mathbf{d}^i) \equiv E[\log f(\mathbf{x}|\mathbf{d})|\mathbf{y}, \mathbf{d}^i]$;

2. Maximization step: Solve $\mathbf{d}^{i+1} = \arg \max_{\mathbf{d}} Q(\mathbf{d}|\mathbf{d}^i)$;

where $\mathbf{d}^i$ is the data bit sequence estimate for the ith iteration. Because the complete data does not actually exist, the algorithm instead maximizes the conditional expectation of the conditional density. Each iteration of the EM algorithm causes the likelihood function to increase and the algorithm will converge in most practical applications. Further discussion on the convergence properties can be found in [16,17]. In order to initialize the algorithm, an initial estimate must be provided. The choice of the initial estimate can significantly affect the outcome of the algorithm. Poorly chosen initial estimates can cause convergence to local maxima.

The EM algorithm can be very useful when applied to channels with undesirable random parameters, $\mathbf{a}$. For example, $\mathbf{a}$ could represent fading coefficients or multipath delays. A natural choice for the complete data would be $\mathbf{x} = (\mathbf{y}, \mathbf{a})$. By making some simple manipulations, the E-step can be expressed in a more desirable form. The

E-step can be rewritten:

$$Q(\mathbf{d}|\mathbf{d}^i) = E[\log p(\mathbf{y}|\mathbf{a},\mathbf{d})|\mathbf{y},\mathbf{d}^i]. \tag{2.11}$$

The advantage of the above expression is that $p(\mathbf{y}|\mathbf{a},\mathbf{d})$ can be very easy to write.

D. RAKE Receiver

The RAKE receiver is an attempt to undo the effects of multipath. Fig. 6 shows the delayed received signal configuration of the RAKE receiver for the given multipath fading channel model. Mathematically, the RAKE receiver can also be expressed as:

$$u = \sum_{m=1}^{M} \int_{-\infty}^{\infty} r(t) a_m^*(t) s^*(t - \tau_m) dt, \tag{2.12}$$

where u is the decision statistic. This expression represents the delayed reference configuration of the RAKE receiver.

At the receiver, the received signal is delayed by several increments until all of the signal paths line up in time. Each of the paths is corrected for fading and then added together coherently. The resulting signal is then correlated with the original transmitted signal to produce the decision statistics.

When $r(t)$ from (2.1) is substituted into the expression for the RAKE receiver (2.12), the following autocorrelation function appears:

$$\int_{-\infty}^{\infty} s(t - \tau_k) s(t - \tau_m) dt. \tag{2.13}$$

If this function has a non-zero value when $k \neq m$, then the receiver will experience interference caused by adding the paths together. If the autocorrelation function is zero when $k \neq m$, then the RAKE receiver is the optimum receiver for the channel. Wideband signals used in CDMA systems usually meet this condition. That is why

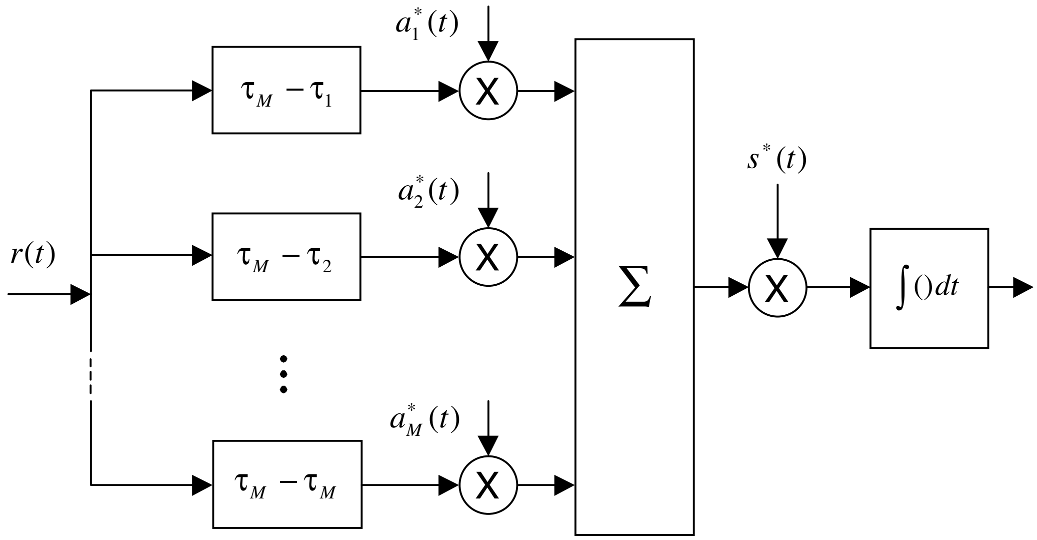

Fig. 6. RAKE receiver (delayed received signal configuration).

the RAKE receiver is used extensively in CDMA. If this condition is not met, the performance of the receiver will be limited by the self-induced interference. A more complicated receiver is necessary to overcome the correlation problem.

CHAPTER III

MULTIPATH RECEIVERS

This chapter shows the mathematical derivations of receivers for the multipath fading channel. The first section defines the channel model. The following sections contain derivations of the optimum (ML) receiver, the EM based receiver, and the RAKE receiver.

A. Channel Model

The first step in deriving the receivers is to define the mathematical model of the received signal. The complex baseband received signal can be written:

$$r(t) = \sum_{k=1}^{P} d_k \sum_{m=1}^{M} a_k^{(m)} p(t - kT - \tau_m) + n(t), \tag{3.1}$$

where k is the bit index, d_k is the kth bit, $a_k^{(m)}$ is the complex fading variable for the mth path and kth bit, $p(t)$ is the unit energy complex baseband pulse shape, T is the time between bits, τ_m is the path delay for the mth path, P is the total number of bits, M is the total number of paths, and $n(t)$ is a white complex Gaussian process.

The received signal can also be represented as a vector of matched filter samples. The matched filter samples from the mth signal path are defined as:

$$r_k^{(m)} = \int_{-\infty}^{\infty} r(t) p^*(t - kT - \tau_m) dt. \tag{3.2}$$

The generalized receiver structure used to produce the received signal samples is shown in Fig. 7.

The matched filter samples from the M signal paths are stacked in a single vector

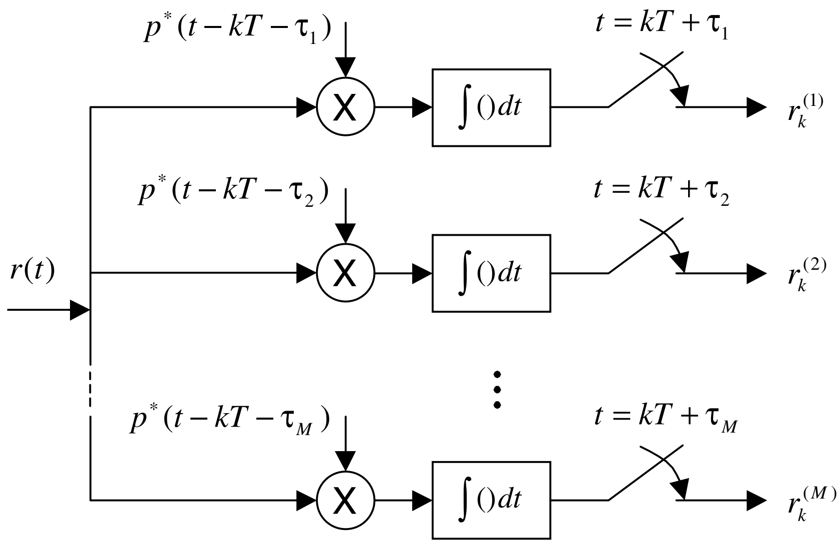

Fig. 7. Generalized receiver structure.

$\mathbf{y}$, defined as:

$$\mathbf{y} = \begin{bmatrix} \mathbf{r}^{(1)} \\ \mathbf{r}^{(2)} \\ \vdots \\ \mathbf{r}^{(M)} \end{bmatrix}. \tag{3.3}$$

For the 2-path model ($M = 2$), the matched filter samples can be written as:

$$\mathbf{y} = \begin{bmatrix} \mathbf{r}^{(1)} \\ \mathbf{r}^{(2)} \end{bmatrix} = \begin{bmatrix} \mathbf{Da}^{(1)} + \mathbf{SDa}^{(2)} + \mathbf{n}^{(1)} \\ \mathbf{S}^{\dagger}\mathbf{Da}^{(1)} + \mathbf{Da}^{(2)} + \mathbf{n}^{(2)} \end{bmatrix}, \tag{3.4}$$

where

$$S_{i,j} = \int_{-\infty}^{\infty} p^*(t - iT - \tau_1)p(t - jT - \tau_2)dt, \tag{3.5}$$

$$n_k^{(m)} = \int_{-\infty}^{\infty} n(t)p^*(t - kT - \tau_m)dt, \tag{3.6}$$

$$\mathbf{D} = \mathrm{diag}\left\{d_1, \ldots, d_P\right\}. \tag{3.7}$$

B. Optimum Receiver

The optimum receiver in the maximum-likelihood sense will be derived in this section. The ML receiver is defined as:

$$\hat{\mathbf{d}} = \arg \max_{\mathbf{d}} p(\mathbf{y}|\mathbf{d}). \tag{3.8}$$

Because $\mathbf{y}$ given $\mathbf{d}$ is just a linear combination of complex Gaussian random variables, it is known that $p(\mathbf{r}|\mathbf{d})$ is a jointly Gaussian distribution. It is only necessary to find the mean and covariance to completely describe the distribution.

The mean vector as a function of $\mathbf{d}$ is defined as:

$$\boldsymbol{\mu}(\mathbf{d}) = E[\mathbf{y}|\mathbf{d}], \tag{3.9}$$

and the covariance matrix as a function of $\mathbf{d}$ is defined as:

$$\mathbf{R}(\mathbf{d}) = E\left[(\mathbf{y} - \boldsymbol{\mu}(\mathbf{d}))(\mathbf{y} - \boldsymbol{\mu}(\mathbf{d}))^\dagger|\mathbf{d}\right], \tag{3.10}$$

where $\dagger$ represents the complex conjugated transpose operation. For jointly Gaussian statistics, the ML receiver is now written as:

$$\hat{\mathbf{d}} = \arg \min_{\mathbf{d}} \left[\mathbf{y} - \boldsymbol{\mu}(\mathbf{d})\right]^\dagger \left[\mathbf{R}(\mathbf{d})\right]^{-1} \left[\mathbf{y} - \boldsymbol{\mu}(\mathbf{d})\right]. \tag{3.11}$$

For the 2-path model ($M = 2$), (3.4) can be substituted into (3.9) and (3.10) to get:

$$\boldsymbol{\mu}(\mathbf{d}) = \mathbf{0}, \tag{3.12}$$

$$\mathbf{R}(\mathbf{d}) = \begin{bmatrix} \mathbf{D}\mathbf{Q}_1\mathbf{D}^* + \mathbf{S}\mathbf{D}\mathbf{Q}_2\mathbf{D}^*\mathbf{S}^\dagger + 2\sigma^2\mathbf{I} & \mathbf{D}\mathbf{Q}_1\mathbf{D}^*\mathbf{S} + \mathbf{S}\mathbf{D}\mathbf{Q}_2\mathbf{D}^* + 2\sigma^2\mathbf{S} \\ \mathbf{S}^\dagger\mathbf{D}\mathbf{Q}_1\mathbf{D}^* + \mathbf{D}\mathbf{Q}_2\mathbf{D}^*\mathbf{S}^\dagger + 2\sigma^2\mathbf{S}^T & \mathbf{S}^\dagger\mathbf{D}\mathbf{Q}_1\mathbf{D}^*\mathbf{S} + \mathbf{D}\mathbf{Q}_2\mathbf{D}^* + 2\sigma^2\mathbf{I} \end{bmatrix}, \tag{3.13}$$

where

$$\mathbf{Q}_m = E\left[\mathbf{a}^{(m)}\mathbf{a}^{(m)\dagger}\right], \tag{3.14}$$

$\mathbf{I}$ is the identity matrix, and σ^2 is the variance of the noise. The optimum receiver for the 2-path model ($M = 2$) becomes:

$$\hat{\mathbf{d}} = \arg\min_{\mathbf{d}} \mathbf{y}^\dagger \left[\mathbf{R}(\mathbf{d})\right]^{-1} \mathbf{y}. \tag{3.15}$$

C. EM Based Receiver

The first step in deriving the EM based receiver for the given multipath fading channel is to choose the complete data. If the fading is known, then the problem of writing the likelihood function becomes much easier. Therefore, we will choose the complete data to be the fading vectors $\mathbf{a}^{(1)}, \ldots \mathbf{a}^{(M)}$ along with the received signal, $r(t)$. The likelihood function of $r(t)$ given $\mathbf{a}^{(1)}, \ldots, \mathbf{a}^{(M)}$ and $\mathbf{d}$ is:

$$p\left(r(t)|\mathbf{d}, \mathbf{a}^{(1)}, \ldots, \mathbf{a}^{(M)}\right) = C \cdot \exp\left\{-\frac{1}{2\sigma^2}\int_{\infty}^{\infty}\left[r(t) - \sum_{k=1}^{P}\sum_{m=1}^{M} d_k a_k^{(m)} p(t - kT - \tau_m)\right] \left[r(t) - \sum_{j=1}^{P}\sum_{l=1}^{M} d_j a_j^{(l)} p(t - jT - \tau_l)\right]^* dt\right\}. \tag{3.16}$$

The log-likelihood function, after some simplification, can be expressed as:

$$\log\left[p\left(r(t)|\mathbf{d},\mathbf{a}^{(1)},\ldots,\mathbf{a}^{(M)}\right)\right] = \Re\left\{\sum_{k=1}^{P}\sum_{m=1}^{M} d_k^* a_k^{(m)*}\int_{-\infty}^{\infty} r(t)p^*(t-kT-\tau_m)dt\right\}$$
$$-\frac{1}{2}\sum_{k=1}^{P}\sum_{m=1}^{M}\sum_{j=1}^{P}\sum_{l=1}^{M} d_k d_j^* a_k^{(m)} a_j^{(l)*}\int_{-\infty}^{\infty} p(t-kT-\tau_m)p^*(t-jT-\tau_l)dt. \quad (3.17)$$

By making some substitutions, (3.17) can be expressed more compactly. After making these substitutions and taking an additional expectation, the expectation step of the EM algorithm can be written as:

$$Q\left(\mathbf{d}|\mathbf{d}^i\right) = \Re\left\{\sum_{k=1}^{P} d_k^* u_k^i\right\} - \frac{1}{2}\sum_{k=1}^{P}\sum_{j=1}^{P} d_k d_j^* x_{k,j}^i, \quad (3.18)$$

where

$$u_k^i = \sum_{m=1}^{M} \hat{a}_k^{(m)i*} r_k^{(m)}, \quad (3.19)$$

$$x_{k,j}^i = \sum_{m=1}^{M}\sum_{l=1}^{M} E\left[a_k^{(m)} a_j^{(l)*}|\mathbf{d}^i, r(t)\right] W_{k,m,j,l}, \quad (3.20)$$

$$\hat{a}_k^{(m)i} = E\left[a_k^{(m)}|\mathbf{d}^i, r(t)\right], \quad (3.21)$$

$$r_k^{(m)} = \int_{-\infty}^{\infty} r(t)p^*(t-kT-\tau_m)dt, \quad (3.22)$$

$$W_{k,m,j,l} = \int_{-\infty}^{\infty} p(t-kT-\tau_m)p^*(t-jT-\tau_l)dt. \quad (3.23)$$

In general, the MMSE estimate of the fading called for in (3.21) is straightforward to write since the fading and the received signal (given the data) are jointly Gaussian.

The maximization step of the EM algorithm is:

$$\arg\max_{\mathbf{d}} Q\left(\mathbf{d}|\mathbf{d}^i\right). \tag{3.24}$$

This maximization can be performed using the Viterbi algorithm after the likelihood function is expressed recursively as:

$$Q\left(\mathbf{d}|\mathbf{d}^i\right)_n = Q\left(\mathbf{d}|\mathbf{d}^i\right)_{n-1} + \Re\left\{d_n^* u_n^i - \frac{1}{2}x_{n,n}^i d_n^* d_n - \sum_{m=n+1}^{n+L} x_{m,n}^i d_n^* d_{n-m}\right\}, \tag{3.25}$$

where $x_{m,n}^i = 0$ for $|n-m| > L$.

For the 2-path model ($M = 2$), the MMSE estimates of the fading are:

$$\hat{\mathbf{a}}^{(1)i} = E\left[\mathbf{a}^{(1)}|\mathbf{d}^i, r(t)\right] = \begin{bmatrix}\mathbf{Q}_1\mathbf{D}_i^* & \mathbf{Q}_1\mathbf{D}_i^*\mathbf{S}\end{bmatrix}\left[\mathbf{R}(\mathbf{d}^i)\right]^{-1}\mathbf{y}, \tag{3.26}$$

$$\hat{\mathbf{a}}^{(2)i} = E\left[\mathbf{a}^{(2)}|\mathbf{d}^i, r(t)\right] = \begin{bmatrix}\mathbf{Q}_2\mathbf{D}_i^*\mathbf{S}^\dagger & \mathbf{Q}_2\mathbf{D}_i^*\end{bmatrix}\left[\mathbf{R}(\mathbf{d}^i)\right]^{-1}\mathbf{y}, \tag{3.27}$$

$$E\left[\mathbf{a}^{(1)}\mathbf{a}^{(1)\dagger}|\mathbf{d}^i, r(t)\right] = \mathbf{Q}_1 - \begin{bmatrix}\mathbf{Q}_1\mathbf{D}_i^* & \mathbf{Q}_1\mathbf{D}_i^*\mathbf{S}\end{bmatrix}\left[\mathbf{R}(\mathbf{d}^i)\right]^{-1}\begin{bmatrix}\mathbf{D}_i\mathbf{Q}_1\\ \mathbf{S}^\dagger\mathbf{D}_i\mathbf{Q}_1\end{bmatrix} + \hat{\mathbf{a}}^{(1)i}\hat{\mathbf{a}}^{(1)i\dagger}, \tag{3.28}$$

$$E\left[\mathbf{a}^{(2)}\mathbf{a}^{(2)\dagger}|\mathbf{d}^i, r(t)\right] = \mathbf{Q}_2 - \begin{bmatrix}\mathbf{Q}_2\mathbf{D}_i^*\mathbf{S}^\dagger & \mathbf{Q}_2\mathbf{D}_i^*\end{bmatrix}\left[\mathbf{R}(\mathbf{d}^i)\right]^{-1}\begin{bmatrix}\mathbf{S}\mathbf{D}_i\mathbf{Q}_2\\ \mathbf{D}_i\mathbf{Q}_2\end{bmatrix} + \hat{\mathbf{a}}^{(2)i}\hat{\mathbf{a}}^{(2)i\dagger}. \tag{3.29}$$

D. RAKE Receiver

The RAKE receiver for the given multipath fading channel makes symbol by symbol decisions based on the statistics u_k:

$$u_k = \sum_{m=1}^{M} \hat{a}_k^{(m)*} r_k^{(m)}. \tag{3.30}$$

For a general linear modulation scheme, the symbol decisions are made by:

$$\hat{d}_k = \arg \max_{d_k} \Re\left\{d_k^* u_k\right\} - \frac{1}{2}\|d_k\|^2. \tag{3.31}$$

For BPSK modulation, the bit decisions are made by:

$$\hat{d}_k = \operatorname{sgn}\left[\Re\left\{u_k\right\}\right]. \tag{3.32}$$

It is clear from (3.31) that the RAKE receiver ignores the intersymbol interference created by the multipath channel. By making symbol by symbol decisions, the RAKE receiver will have very low complexity but will suffer considerably in performance.

The fading variables in (3.30) are assumed to be estimated by other methods. For this study, the RAKE receiver will use perfect channel estimates. In this way, a lower bound to the RAKE receiver's performance will be achieved.

CHAPTER IV

SIMULATION IMPLEMENTATION DETAILS

This chapter contains the details that are necessary to create the simulations. The first section describes the channel and how to create the random processes for the channel. The second section discusses the transmitted signal format and the message structure. The final sections describe the specific implementations of each receiver.

It should be noted that this simulation uses complex baseband signals. By doing so, all of the information of a bandpass signal is retained without having to sample at a rate above the carrier frequency. The real axis of the complex baseband signal represents the information on the inphase carrier while the imaginary axis represents the information on the quadrature carrier.

A. Channel

In the real world, the signals transmitted across the channel are continuous time waveforms. Because the simulations are performed on a computer, the waveforms must be represented by discrete time samples. In general, the waveforms must be sampled at least at the Nyquist rate for all the signal information to be retained. For these simulations, a sample rate of 50 samples per symbol is used. Assuming that square pulses or other narrow-band pulses are used, the given sampling rate is ample. Another ramification of the sampling rate is that the minimum multipath delay this simulation can handle is 1/50th of a symbol interval.

The number of paths in the channel model has an impact on the complexity of the receiver. For simplicity, only the two path model ($M = 2$) will be simulated. Also, the path delay, τ, will be limited to one symbol interval. Within the symbol interval, up to 50 different path delays can be simulated.

1. Rayleigh Fading Simulator

The Rayleigh fading process is a series of correlated complex Gaussian random variables completely defined by their mean, variance, and autocorrelation. The fading process for a given path is independent of all the other paths. The statistics of each path are:

$$E\left[a_k^{(m)}\right] = 0, \tag{4.1}$$

$$E\left[a_k^{(m)} a_k^{(m)*}\right] = 1, \tag{4.2}$$

$$E\left[a_k^{(m)} a_n^{(m)*}\right] = J_0\left[2\pi f_d T(k-n)\right]. \tag{4.3}$$

The variance of the fading is chosen to be one so that the average energy per bit of the received signal equals the average energy per bit of the transmitted signal. Also, all of the paths will have the same average energy.

The fading process is simulated using a frequency domain method [24]. The first step is to generate $2N+1$ independent complex Gaussian random variables with

$$E\left[x_n\right] = 0, \tag{4.4}$$

$$E\left[|x_n|^2\right] = \sigma_n^2 = \frac{\alpha}{\sqrt{f_d^2 - \left(\frac{n}{T_a}\right)^2}}, \tag{4.5}$$

$$\alpha = \left[\sum_{n=-N}^{N} \frac{1}{\sqrt{f_d^2 - \left(\frac{n}{T_a}\right)^2}}\right]^{-1}, \tag{4.6}$$

where T_a is the total time duration of the fading process. The parameter N is chosen by

$$N = \lfloor f_d T_a \rfloor. \tag{4.7}$$

The final step is to take a discrete Fourier transform (DFT) of the x_n's to get the fading process samples.

$$a_k^{(m)} = \sum_{n=-N}^{N} x_n \left(e^{j2\pi \frac{T}{T_a}} \right)^{kn}, \tag{4.8}$$

where T is the symbol interval.

2. Gaussian Noise and SNR

The complex additive white Gaussian noise (AWGN) process, $n(t)$, has zero mean and variance per dimension

$$\sigma^2 = \frac{N_0}{2}, \tag{4.9}$$

where N_0 is the power spectral density of the noise. For this simulation, the noise component of the complex matched filter samples takes the form

$$n = n^{(I)} + jn^{(Q)}, \tag{4.10}$$

where $n^{(I)}$ and $n^{(Q)}$ are drawn independently from the distribution $\mathcal{N}(0, N_0/2)$.

The signal to noise ratio (SNR) is defined as the average energy per bit divided by the power spectral density of the noise.

$$\text{SNR} = \frac{E_b}{N_0}. \tag{4.11}$$

The SNR is difficult to define for the multipath fading channel. For this simulation, SNR will be defined in a similar manner to a two-path diversity channel. E_b will be computed by adding the average received energy per bit of the two signal paths.

If the fading statistics satisfy $E[aa^*] = 1$ for both paths, then E_b will be twice the average energy of the transmitted signal. The average energy of the transmitted signal is determined by the pulse shape, modulation format, and packet structure. SNR is usually the independent variable for the simulations. Once SNR and E_b are determined, N_0 can be computed and the noise samples can be generated accordingly.

B. Transmitted Signal

For this simulation, binary phase shift keying (BPSK) with square pulses will be used. The transmitted signal is of the form

$$s(t) = \sum_k d_k p(t - kT), \tag{4.12}$$

where $d_k \in \{-1, 1\}$ is the transmitted bit and $p(t)$ is a square pulse with duration T and amplitude $1/\sqrt{T}$. The resulting transmitted signal will have a unit average energy per bit. For simplicity, the bit interval T is set to one second. It should be noted that when this signal is sampled at a rate of 50 samples per second (for simulation purposes), the average energy per bit increases by 50 times in the discrete domain. This can be accounted for by dividing the pulse shape $p(t)$ by $\sqrt{50}$.

In order to estimate the fading coefficients, the receiver needs to be provided additional information. Fixing the first bit in each subpacket will allow the receiver to get an initial fading estimate. Subpackets are put together to form a packet and packets are put together to form the entire message. To reduce the complexity of the sequence estimators, receivers will only demodulate one packet at a time. This block processing reduces the decoding delay.

The message structure is shown in Fig. 8. The first bit of each subpacket is $d_k = 1$ and is referred to as a *pilot symbol* [25]. The number of bits in a subpacket is

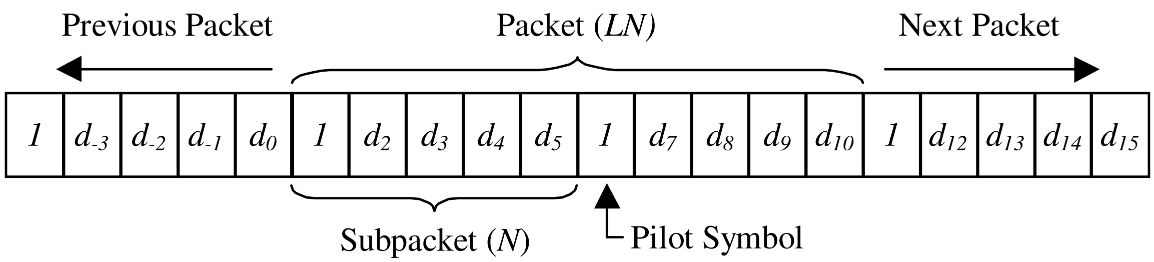

Fig. 8. Message structure for $N = 5$ and $L = 2$.

N and the number of subpackets in a packet is L. The packet length is equal to LN. Because each packet is followed by another packet, the block processing receivers will also include the first pilot symbol of the next packet.

The quantity $1/N$ is called the pilot symbol *insertion rate*, and N is called the pilot symbol *insertion period*. Because only $N-1$ out of N bits actually carries information, the SNR must be penalized by a factor of $N/(N-1)$. It is therefore beneficial to minimize the insertion rate. However, the insertion rate should be high enough to produce reliable fading estimates. There exists an optimum tradeoff between SNR penalty and fading estimation quality which will be determined by simulation. It can be expected that the insertion rate should at least satisfy the Nyquist sampling rate for the fading process. For the fading process, the Nyquist sampling rate is twice the maximum normalized Doppler shift, $f_d T$. Therefore,

$$\frac{1}{N} \geq 2 f_d T. \tag{4.13}$$

C. Optimum Receiver

The optimum receiver implements (3.15). The block length P is the current packet length plus the pilot symbol from the next packet, $P = NL + 1$. The receiver is assumed to be perfectly synchronized with the incoming packet. To obtain the matched filter samples in discrete time, the integrals are simply changed to summations. The likelihood function must be recomputed for every possible bit sequence. Note that the pilot symbols remain fixed during this exhaustive search. At the end, the bit sequence with the lowest resulting likelihood function becomes the final bit sequence decision.

D. EM Based Receiver

In order to implement the EM receiver, an initial estimate of the fading must be provided. The pilot symbols provide a means to initially estimate the fading process. The true MMSE estimate of the fading would require exhaustively iterating through every possible bit combination. In order to significantly reduce the complexity of the algorithm, the linear MMSE estimate of the fading given the pilot symbols and the received signal will be used to initialize the algorithm. The linear MMSE estimate is given by:

$$\hat{\mathbf{a}}^{(m)} = \mathbf{C}_{\mathbf{a}^{(m)},\mathbf{y}} \mathbf{C}_{\mathbf{y},\mathbf{y}}^{-1} \mathbf{y}, \tag{4.14}$$

where

$$\mathbf{C}_{\mathbf{y},\mathbf{y}} = \begin{bmatrix} \mathbf{G}_1 + \mathbf{S}\mathbf{G}_2\mathbf{S}^{\dagger} + 2\sigma^2\mathbf{I} & \mathbf{G}_1\mathbf{S} + \mathbf{S}\mathbf{G}_2 + 2\sigma^2\mathbf{S} \\ \mathbf{S}^{\dagger}\mathbf{G}_1 + \mathbf{G}_2\mathbf{S}^{\dagger} + 2\sigma^2\mathbf{S}^{\mathrm{T}} & \mathbf{S}^{\dagger}\mathbf{G}_1\mathbf{S} + \mathbf{G}_2 + 2\sigma^2\mathbf{I} \end{bmatrix}, \tag{4.15}$$

$$\mathbf{G}_m = E\left[\mathbf{D}\mathbf{a}^{(m)}\mathbf{a}^{(m)\dagger}\mathbf{D}^*\right], \tag{4.16}$$

$$\mathbf{C}_{\mathbf{a}^{(1)},\mathbf{y}} = \begin{bmatrix} \mathbf{Q}_1 E\left[\mathbf{D}^*\right] & \mathbf{Q}_1 E\left[\mathbf{D}^*\right]\mathbf{S} \end{bmatrix}, \tag{4.17}$$

$$\mathbf{C}_{\mathbf{a}^{(2)},\mathbf{y}} = \begin{bmatrix} \mathbf{Q}_2 E\left[\mathbf{D}^*\right]\mathbf{S}^\dagger & \mathbf{Q}_2 E\left[\mathbf{D}^*\right] \end{bmatrix}. \tag{4.18}$$

The matrix $E[\mathbf{D}^*]$ consists of all zeros except for 1's every N positions along the main diagonal. The matrix $E\left[\mathbf{D}\mathbf{a}^{(m)}\mathbf{a}^{(m)\dagger}\mathbf{D}^*\right]$ consists of the main diagonal of $\mathbf{Q}_m$ plus any element of $\mathbf{Q}_m$ where the indices i and j are known symbol positions, $i \neq j$.

The EM receiver can be summarized in the following steps:

1. Given the received signal and the pilot symbols, compute the initial linear MMSE estimate of the fading using (4.14).

2. Given the fading estimate, maximize (3.25) with the Viterbi algorithm.

3. Given the decoded bit sequence, compute the MMSE estimate of the fading with (3.26).

4. Go to step 2 and repeat until the decoded bit sequence does not change (convergence).

E. RAKE Receiver

The RAKE receiver implements (3.32). For this simulation, the RAKE receiver is provided perfect fading estimates in order to produce a lower bound. Because there is no need to estimate the fading, there are no pilot symbols and no packet structure. The simulation produces one long data stream and the RAKE receiver makes symbol by symbol decisions until a predetermined number of errors has occurred.

CHAPTER V

PERFORMANCE COMPARISONS

This chapter contains the simulation plots and a discussion of the results. The first section compares the multipath channel to diversity channels. The second section compares the optimum and EM receivers. The third section compares the RAKE and EM receivers. The final section determines the effect of the packet length and structure on the performance of the EM receiver.

A. Multipath Channel vs. Diversity Channels

In this section, the multipath fading channel is compared to independent diversity channels in Rayleigh fading. The probability of error for a BPSK system with L_d independent diversity channels in known Rayleigh fading is given by:

$$P(e) = \left[\frac{1}{2}(1-\mu)\right]^{L_d} \sum_{k=0}^{L_d-1} \binom{L_d - 1 + k}{k} \left[\frac{1}{2}(1+\mu)\right]^{k}, \tag{5.1}$$

where

$$\mu = \sqrt{\frac{\frac{\mathrm{SNR}_b}{L_d}}{1 + \frac{\mathrm{SNR}_b}{L_d}}}, \tag{5.2}$$

and SNR_b is the signal to noise ratio per bit.

Fig. 9 shows the performance of a two channel diversity system, a single channel system, and the "genie" bounds for two cases of the two path multipath channel. The "genie" bound represents the optimum receiver performance when the fading is known exactly. The results show that the two path multipath channel is similar in performance to a two channel diversity system.

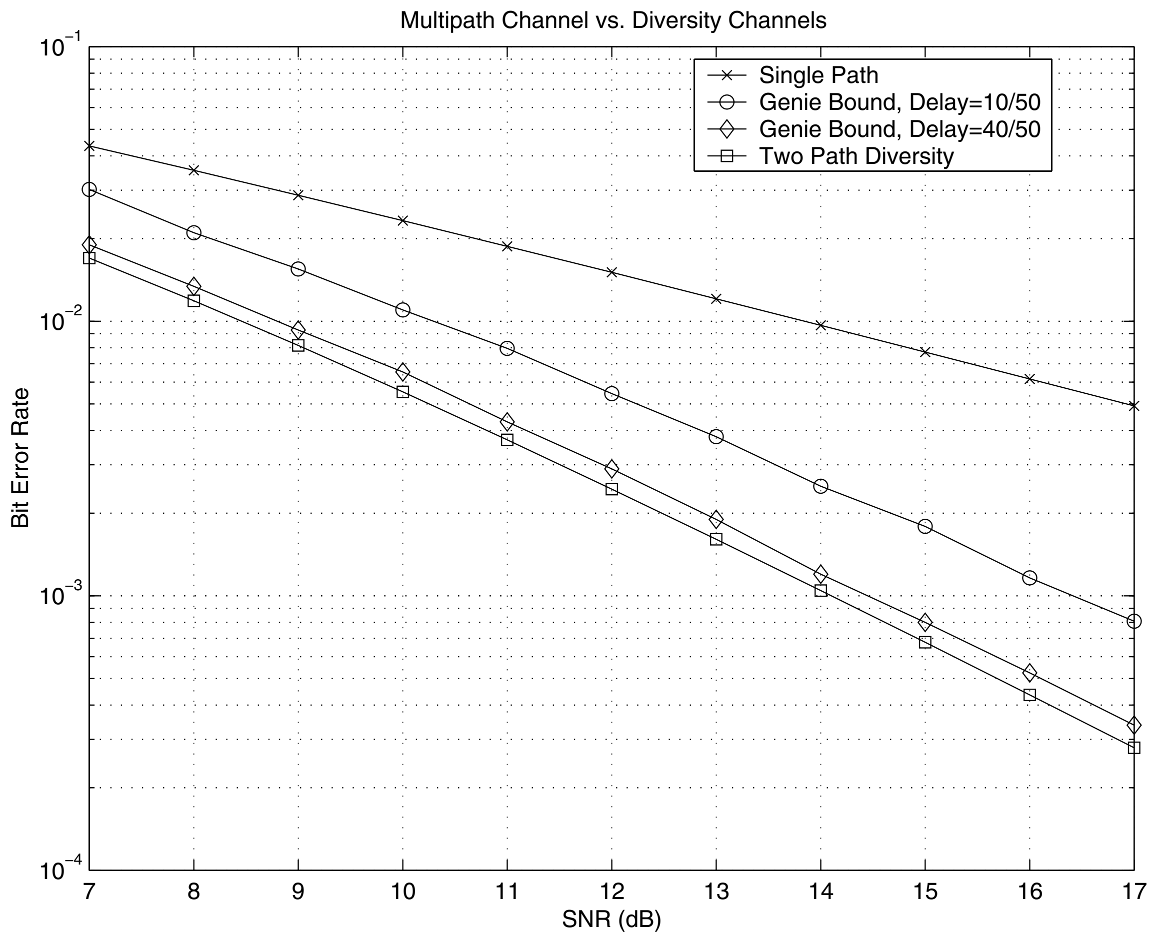

Fig. 9. Multipath channel vs. diversity channels.

B. Performance of Optimum Receiver vs. EM Receiver

In this section, both the bit error rate performance and complexity are used to compare the optimum and EM receivers. The optimum receiver mainly serves as a performance benchmark because its complexity prohibits its practical implementation.

1. Bit Error Rate Performance

For these simulations, the bit error rate performance of the optimum receiver and the EM receiver are compared under the same conditions. Each simulation uses a packet that contains two subpackets of length 5 ($N = 5$, $L = 2$). The packet length is 10 symbols, and the receivers consider a block of 11 symbols after including the first pilot symbol of the next packet. The two-path model is used and the path delay is fixed to 20 samples out of 50 samples in a bit interval ($\tau = 0.4T$). The Doppler shift of the fading ($f_d T$) is changed for each plot to show its effects. The plots contain bit error rates vs. SNR for the optimum receiver, the EM receiver, and the "genie" bound. The "genie" bound represents the optimum receiver performance when the fading is known exactly. Pilot symbols are not used for the "genie" bound.

Figs. 10 through 15 and Tables I through IV show the simulation results. The results show that there can be a significant performance difference (over 2dB) between the optimum and EM receivers at very high fading rates. At lower fading rates, the difference becomes less than a dB. Fig. 15 shows that the slope of the EM receiver's BER curve follows the "genie" bound. This implies that the EM receiver is capable of diversity-like performance when compared to a single path model.

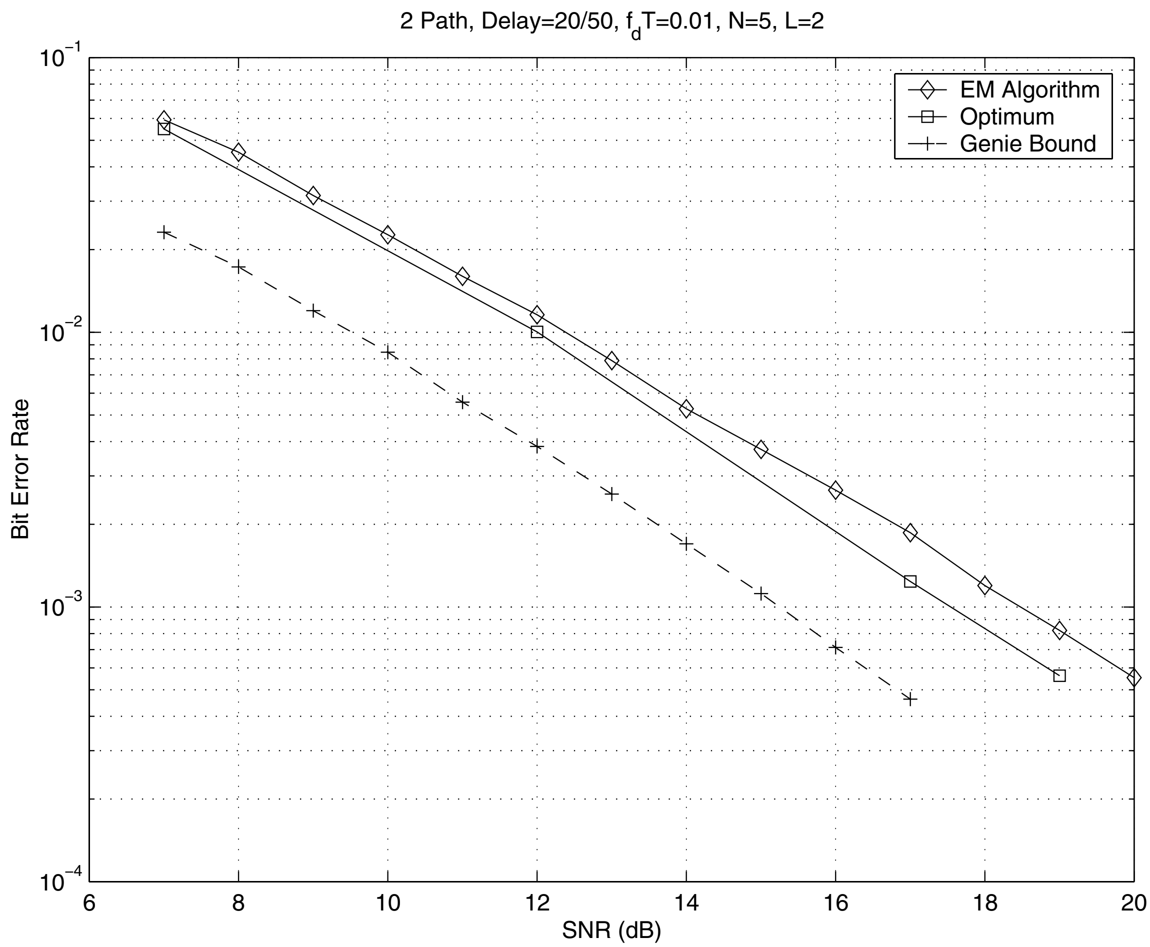

Fig. 10. Bit error rate of optimum and EM receivers ($f_d T = 0.01$).

Table I. Summary of results from Fig. 10.

BER	Opt→EM	Genie→Opt	Genie→EM
10^{-2}	0.4 dB	2.5 dB	2.9 dB
10^{-3}	0.9 dB	2.3 dB	3.2 dB

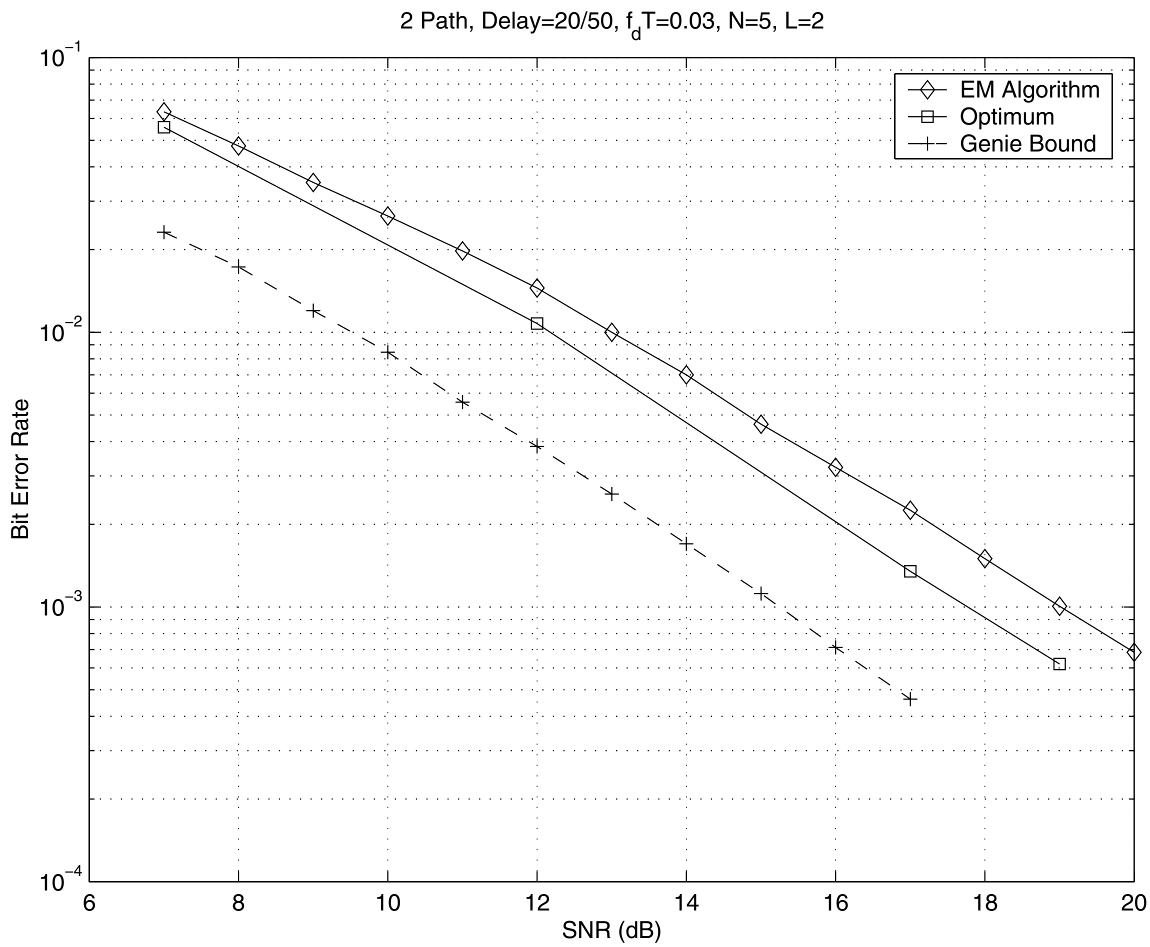

Fig. 11. Bit error rate of optimum and EM receivers ($f_d T = 0.03$).

Table II. Summary of results from Fig. 11.

BER	Opt→EM	Genie→Opt	Genie→EM
10^{-2}	0.8 dB	2.6 dB	3.4 dB
10^{-3}	1.2 dB	2.5 dB	3.7 dB

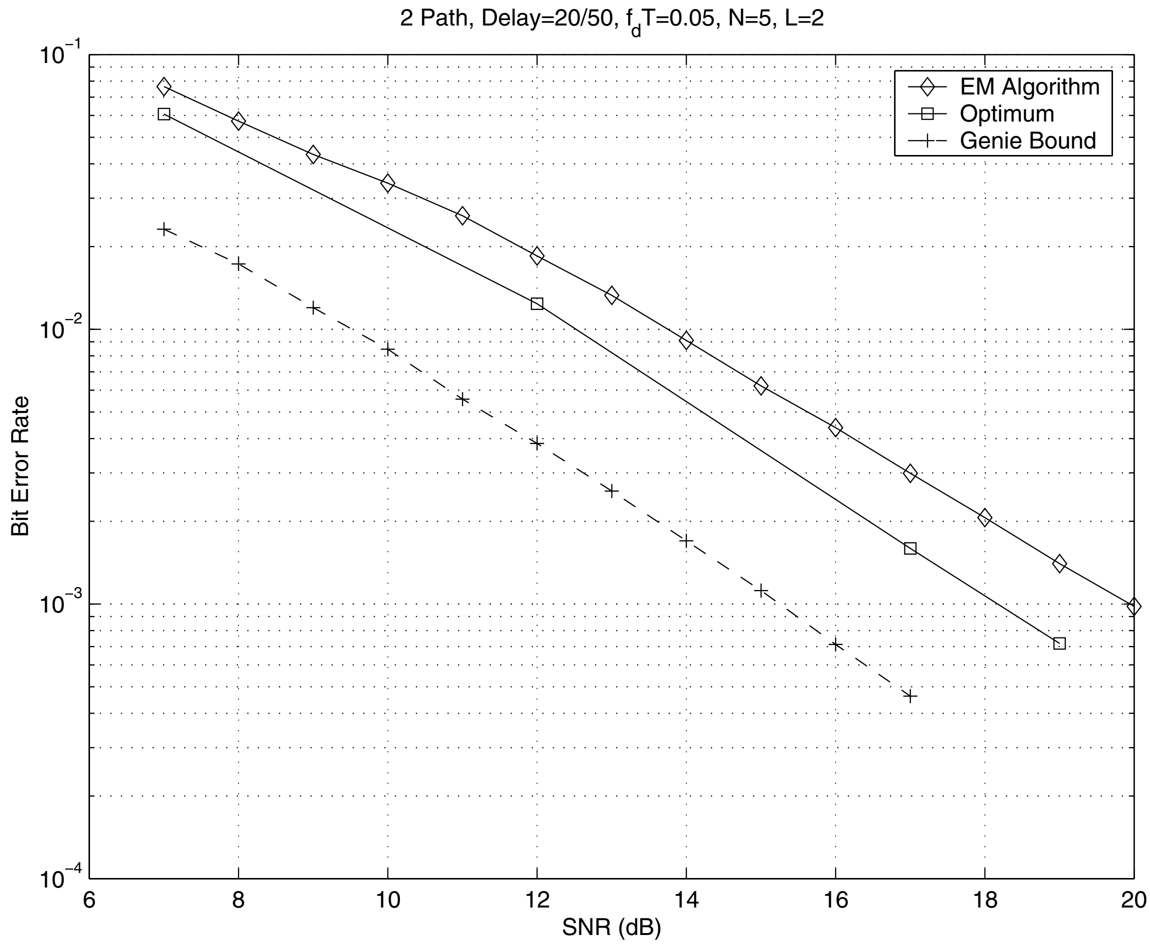

Fig. 12. Bit error rate of optimum and EM receivers ($f_d T = 0.05$).

Table III. Summary of results from Fig. 12.

BER	Opt→EM	Genie→Opt	Genie→EM
10^{-2}	1.2 dB	3.0 dB	4.2 dB
10^{-3}	1.8 dB	2.9 dB	4.7 dB

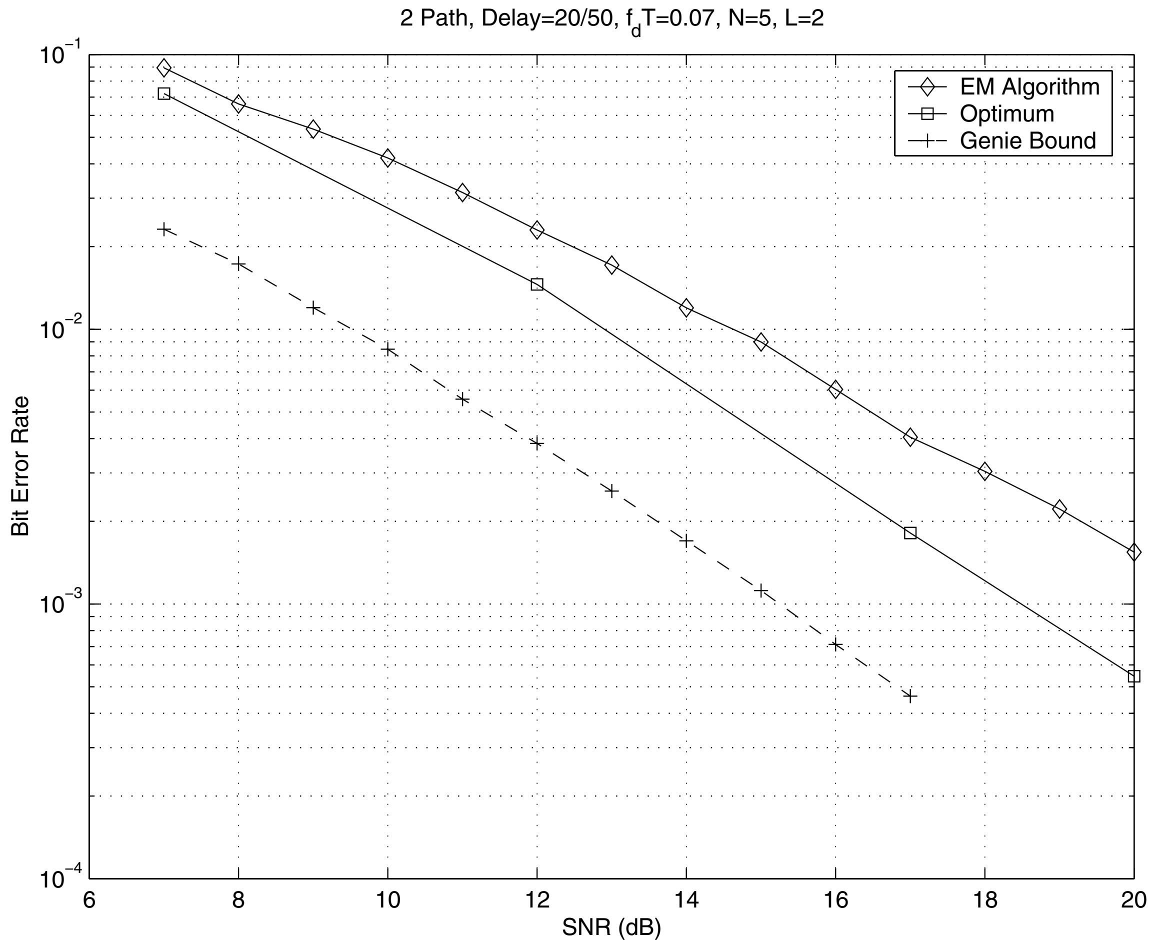

Fig. 13. Bit error rate of optimum and EM receivers ($f_dT = 0.07$).

Table IV. Summary of results from Fig. 13.

BER	Opt→EM	Genie→Opt	Genie→EM
10^{-2}	1.7 dB	3.3 dB	5.0 dB
10^{-3}	2.6 dB	3.2 dB	5.8 dB

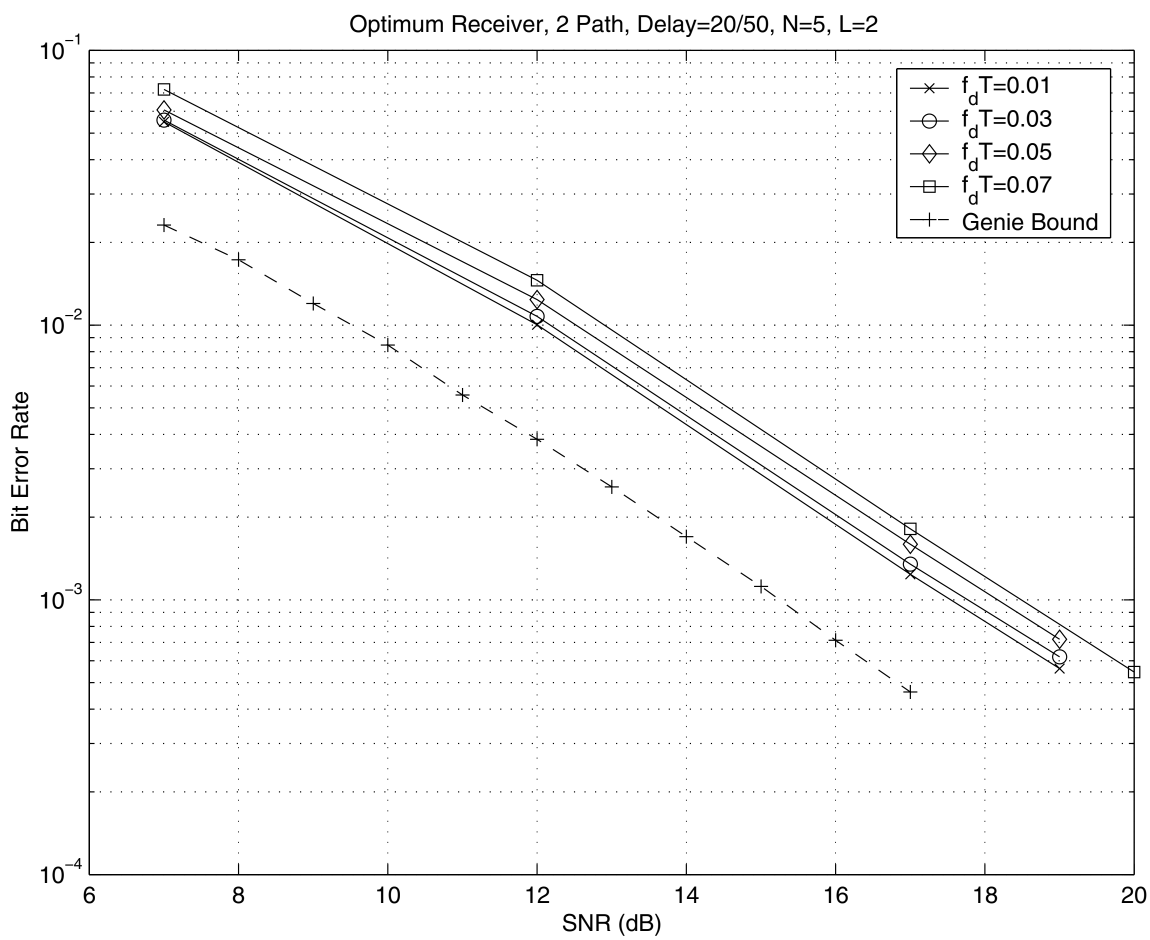

Fig. 14. Bit error rate of optimum receiver (varying f_dT).

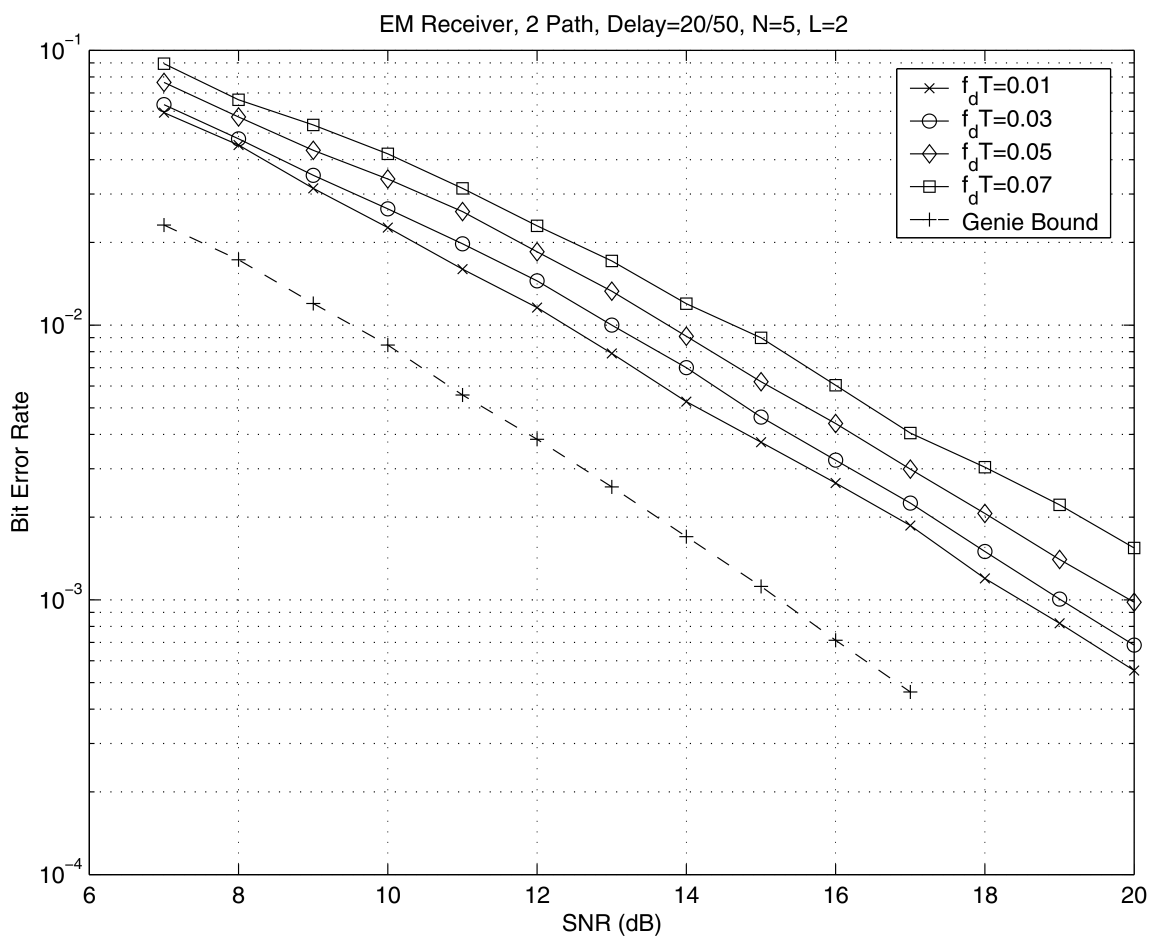

Fig. 15. Bit error rate of EM receiver (varying $f_d T$).

2. Complexity

The EM receiver uses an iterative algorithm. In most cases, the EM algorithm converges within two iterations. Table V shows the average number of iterations required for convergence for various packet lengths.

The complexity of the receivers can be determined by counting the number of floating point operations (flops) that were performed while processing a single data packet and then dividing by the number of bits in a packet. The number of flops per packet were determined using the FLOPS counter in MATLAB. When using higher packet lengths with the optimum receiver, it becomes impossible to experimentally count the flops. Instead these results are extrapolated from the data that could be measured and provided only as a rough comparison to the EM receiver. It should also be noted that the complexity results computed by MATLAB will not be exactly the same as the complexity of these receivers when implemented on a DSP.

Table VI shows the flops per bit required to run the EM and optimum receivers for various packet lengths. According to the results, the EM based receiver has a complexity of $O(N_L^3)$ where N_L is the packet length, and the optimum receiver has a complexity of $O(2^{N_L})$. The optimum receiver quickly becomes impractical for long packet lengths. The EM receiver is capable of processing much longer packet lengths for a given number of flops.

Table V. Average number of iterations required for EM algorithm to converge.

Packet Length	Average Iterations
10	1.07
20	1.19
40	1.28
60	1.48
80	1.54

Table VI. Number of floating point operations (flops) per bit for EM and optimum receivers.

Packet Length	EM Receiver	Optimum Receiver
10	8.47×10^4 flops/bit	2.95×10^6 flops/bit
20	3.07×10^5 flops/bit	1.51×10^9 flops/bit
40	1.15×10^6 flops/bit	7.93×10^{14} flops/bit
60	2.77×10^6 flops/bit	5.53×10^{20} flops/bit
80	4.99×10^6 flops/bit	4.35×10^{26} flops/bit

C. Performance of RAKE Receiver vs. EM Receiver

For these simulations, the RAKE receiver and the EM receiver are compared under unequal conditions. The RAKE receiver is allowed perfect knowledge of the fading without pilot symbols in order to create a lower bound on its performance. Because the EM receiver needs to estimate the fading, its simulation uses a packet that contains two subpackets of length 5 ($N = 5$, $L = 2$). The packet length was 10 symbols, and the receiver considers a block of 11 symbols after including the first pilot symbol of the next packet. For both receiver simulations, the Doppler shift of the fading is set to $f_d T = 0.01$. The multipath delay is changed for each plot to show the effects of different signal correlations on the receivers.

Figs. 16 through 22 show the simulation results. The results show that at high SNR, the real EM receiver will perform better than the best case RAKE receiver. The RAKE receiver appears to have an error floor due to the self-noise caused by neglecting the ISI. The EM receiver takes into account the ISI and does not have an error floor for the bit error rates simulated. The change of the multipath delay has an interesting effect on performance. As the multipath delay is increased, the "genie" bound's performance improves, but the RAKE's performance gets worse. It can be concluded that as the multipath delay increases, the penalty of ignoring the ISI is increased. However, it is easier to undo the negative effects of the ISI. The effect of changing the delay has a more unusual effect on the EM receiver. The performance first gets better and then gets worse. A possible explanation would be that the fading might be easier to estimate when the symbols from one path are split evenly between the symbols from the other path ($\tau = 0.5T$).

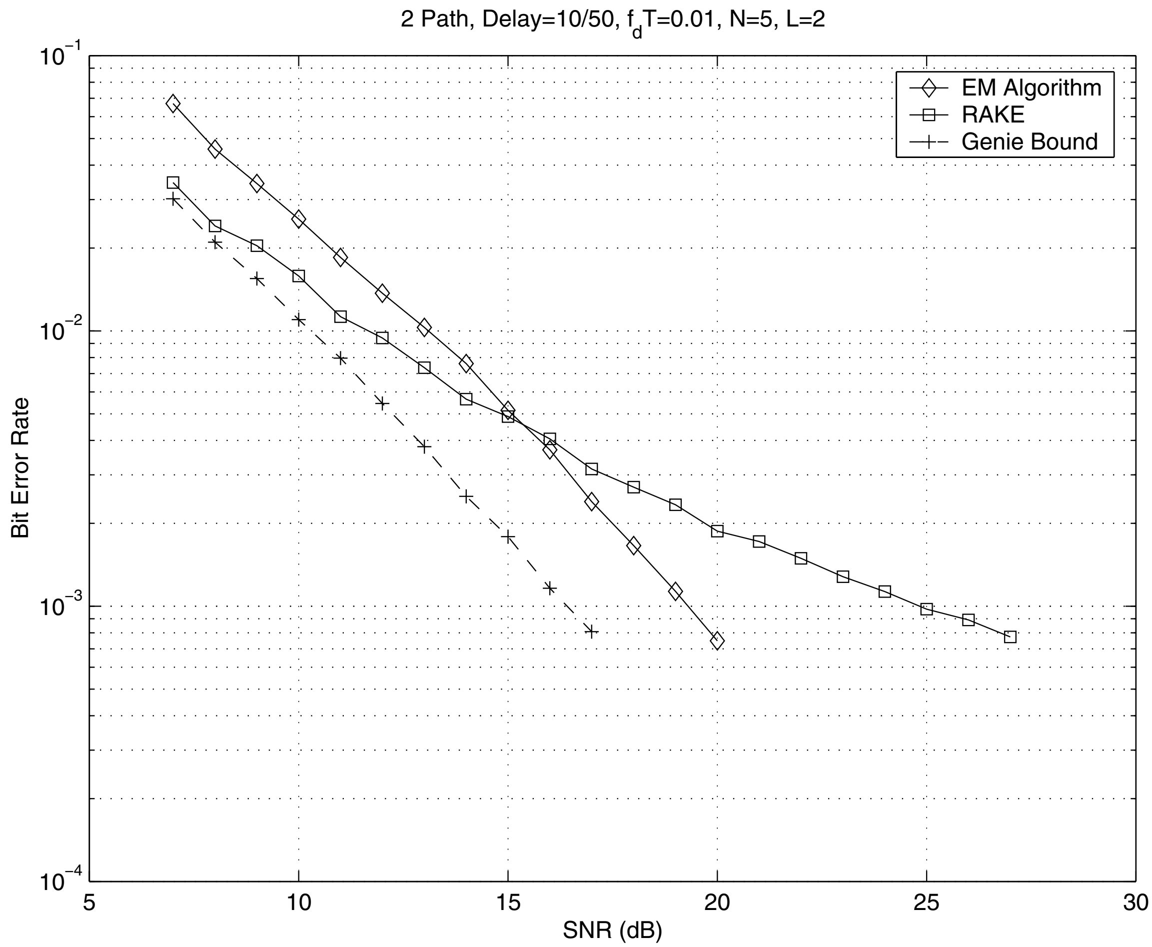

Fig. 16. Bit error rate of RAKE and EM receivers (delay = 10/50).

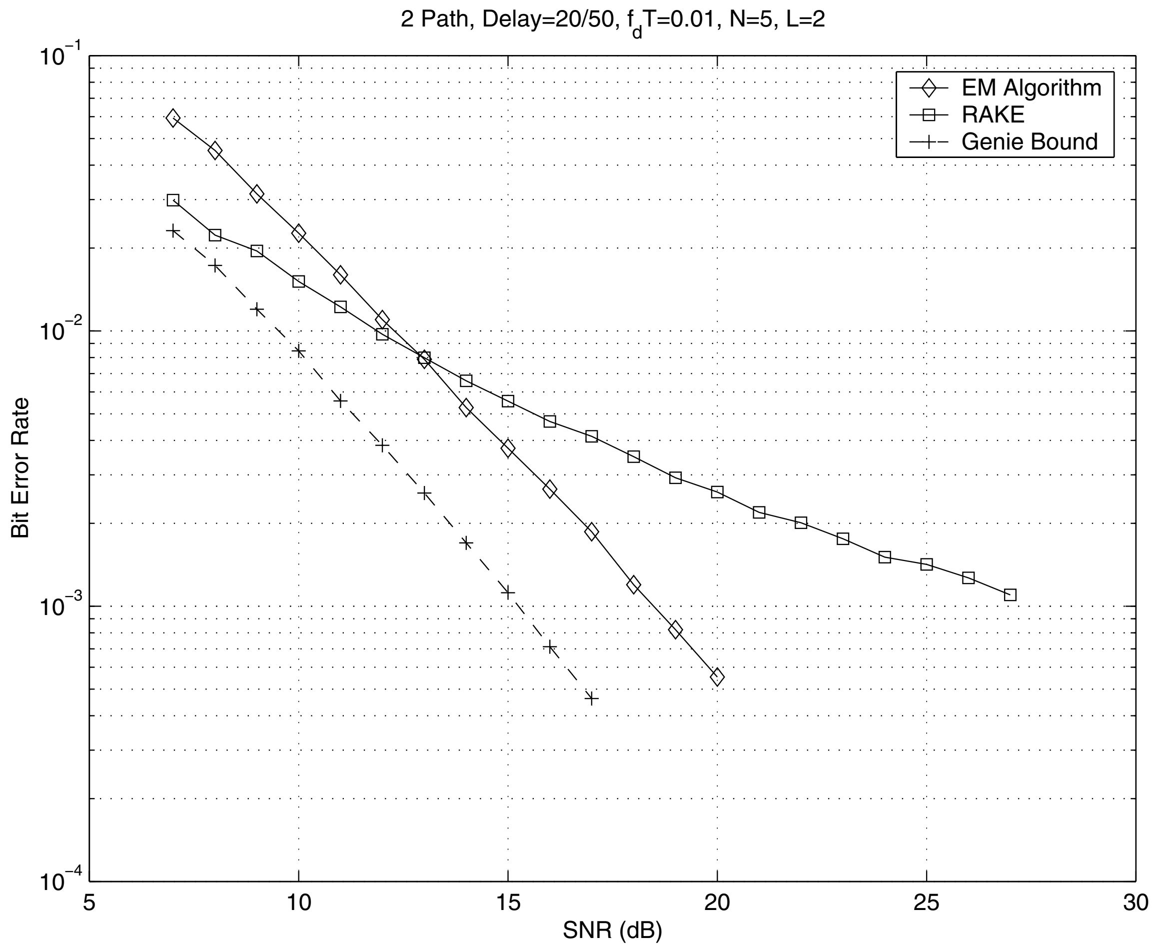

Fig. 17. Bit error rate of RAKE and EM receivers (delay = 20/50).

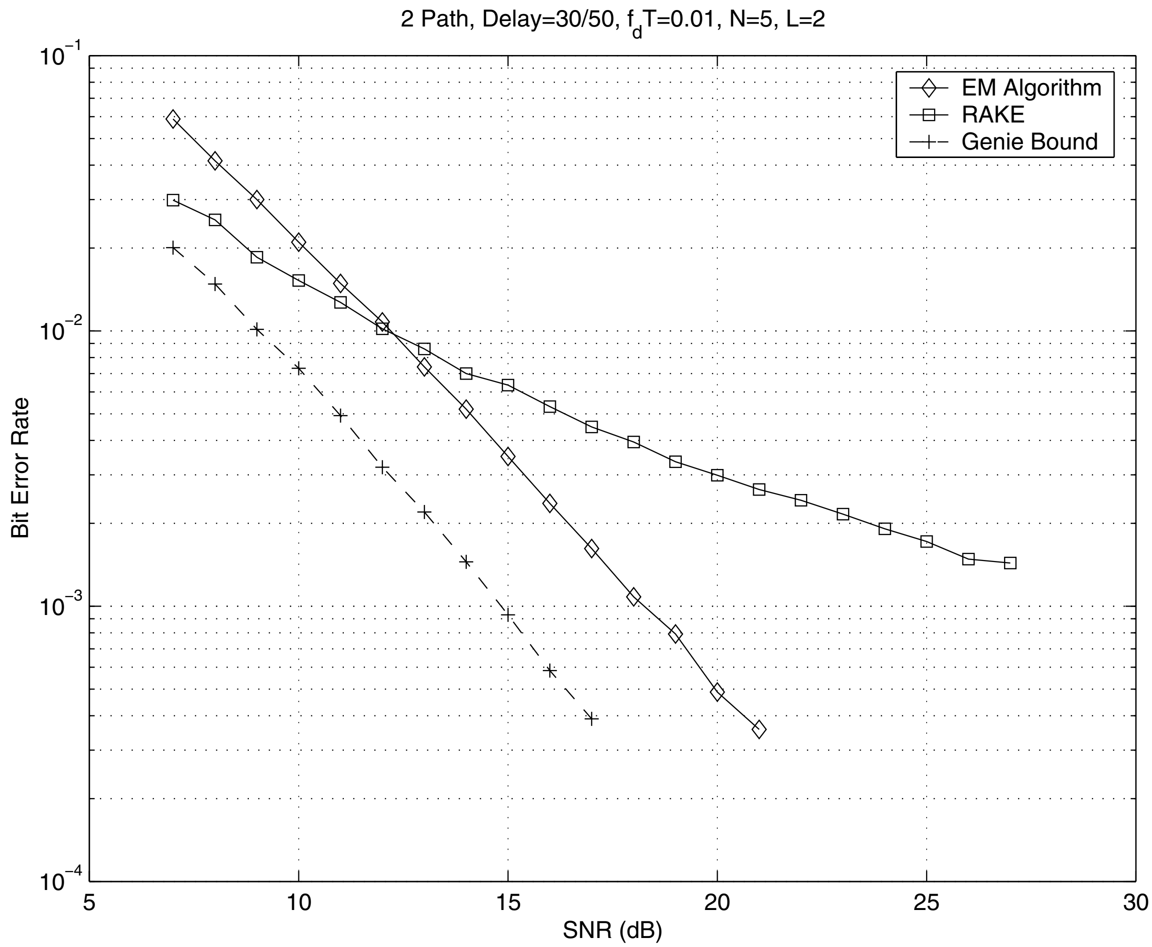

Fig. 18. Bit error rate of RAKE and EM receivers (delay = 30/50).

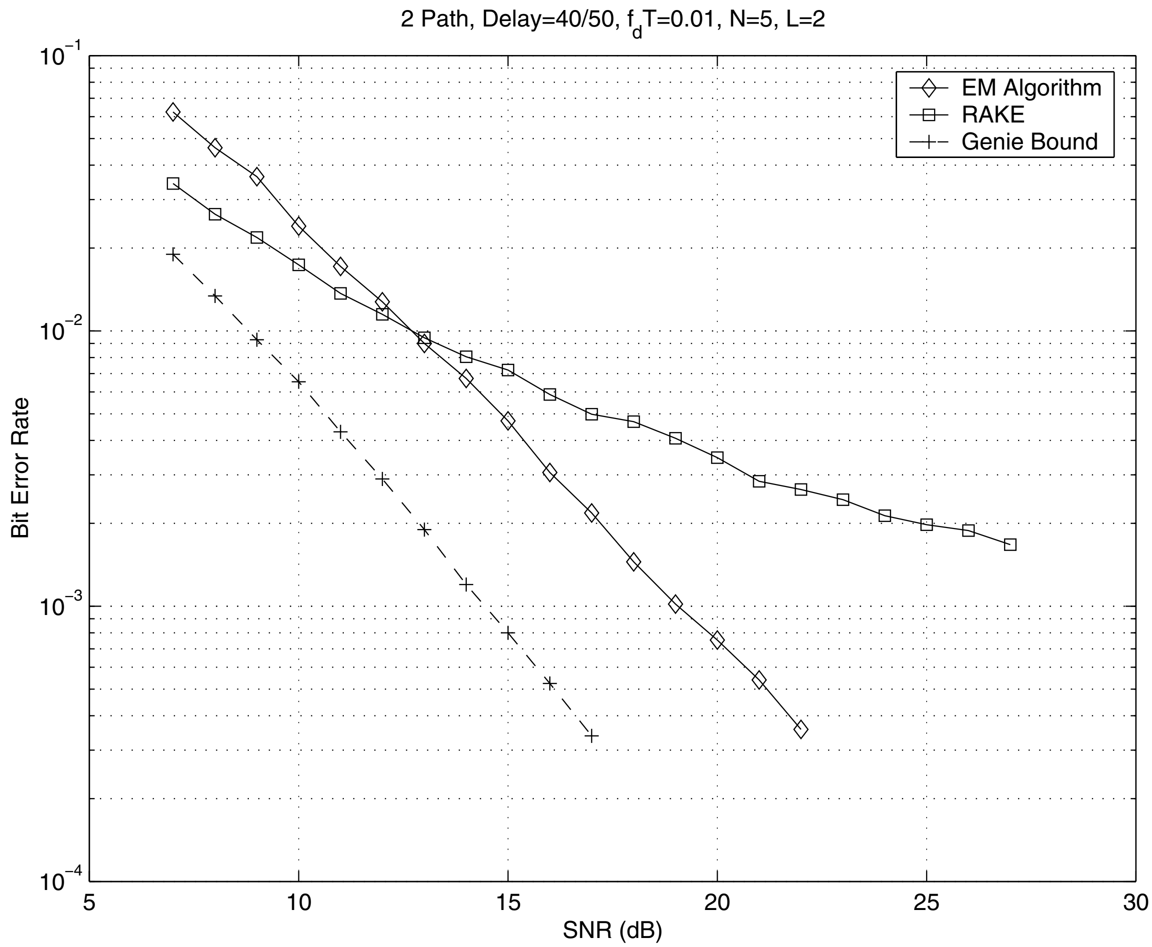

Fig. 19. Bit error rate of RAKE and EM receivers (delay = 40/50).

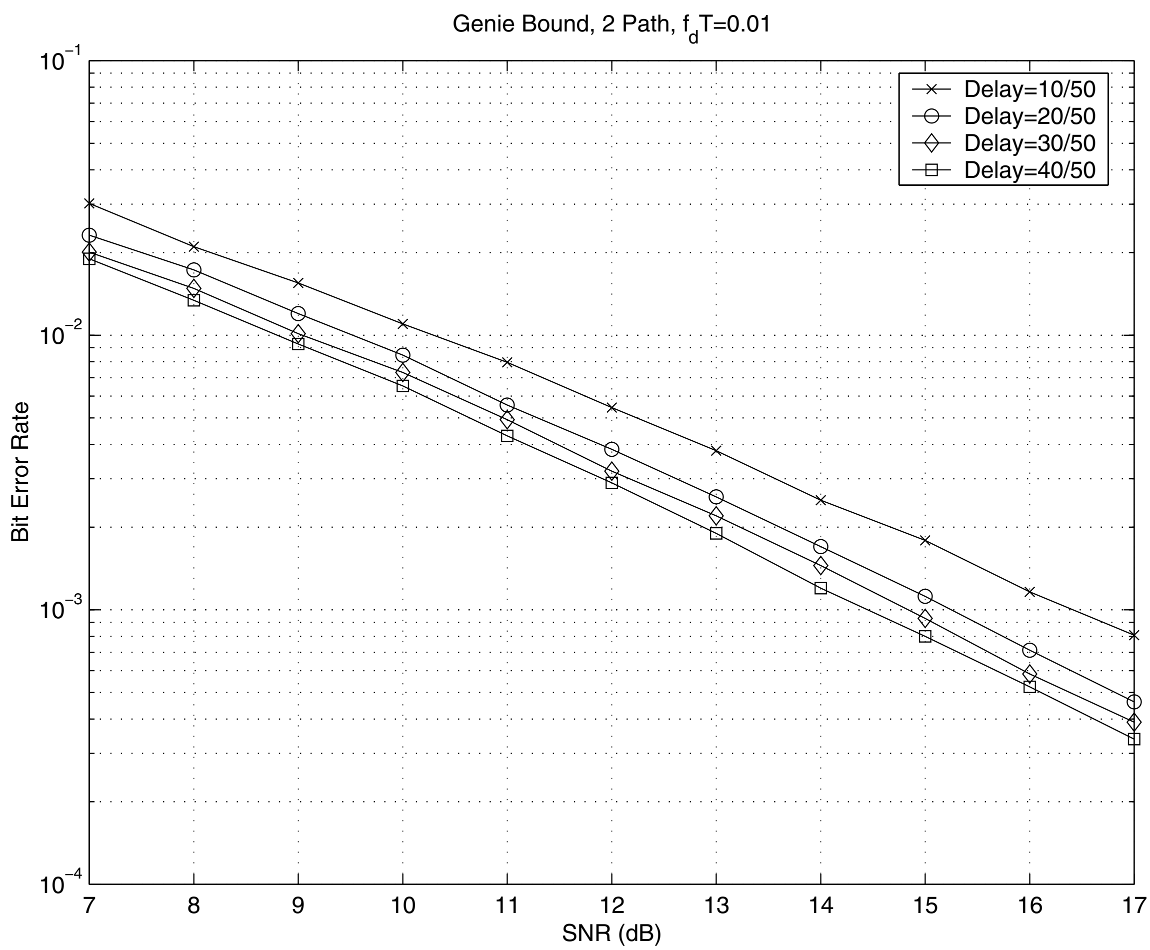

Fig. 20. Bit error rate of "genie" bound (varying delay).

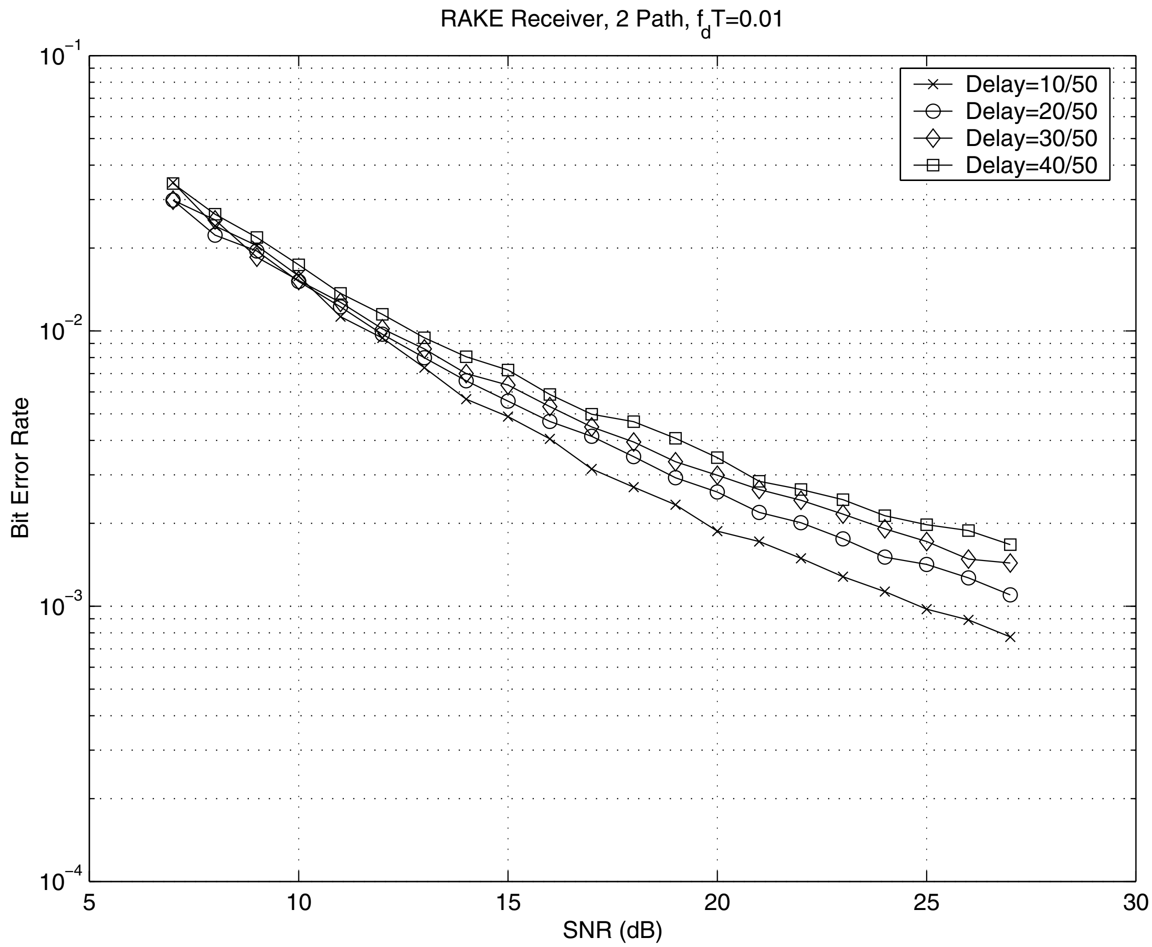

Fig. 21. Bit error rate of RAKE receiver (varying delay).

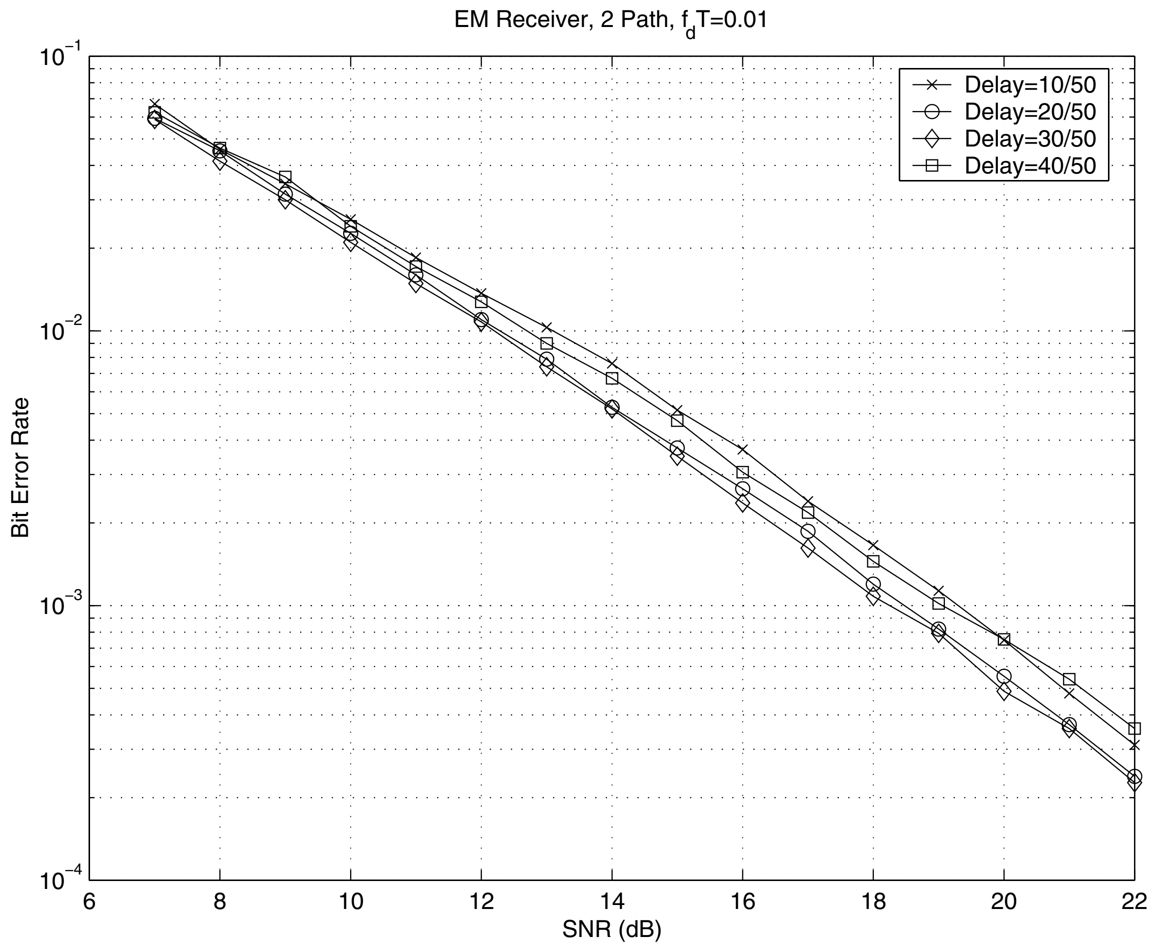

Fig. 22. Bit error rate of EM receiver (varying delay).

D. EM Receiver Performance for Various Packet Structures

In this section, both the bit error rate (BER) of the data and the mean square error (MSE) of the fading estimates is considered for the EM receiver using various packet lengths and structures. The results can help to determine the optimum packet structure for a given set of conditions.

1. Bit Error Rate Performance

For these simulations, the bit error rate performance of the EM receiver is considered for different packet lengths and packet structures. Each simulation uses a path delay of 20 samples out of 50 samples in a bit interval ($\tau = 0.4T$) and the Doppler shift is set to $f_d T = 0.01$. The first simulation considers the effect of packet length with a fixed pilot symbol insertion period ($N = 10$). The next simulations consider the effect of different insertion periods with fixed packet lengths.

Figs. 23 through 26 and Table VII show the simulation results. Fig. 23 shows that the performance of the EM receiver approaches the "genie" bound as the packet length increases. Figs. 24 through 26 show how the packet structure affects performance. Each plot considers a different packet length. In each plot, there is a packet structure that optimizes the performance.

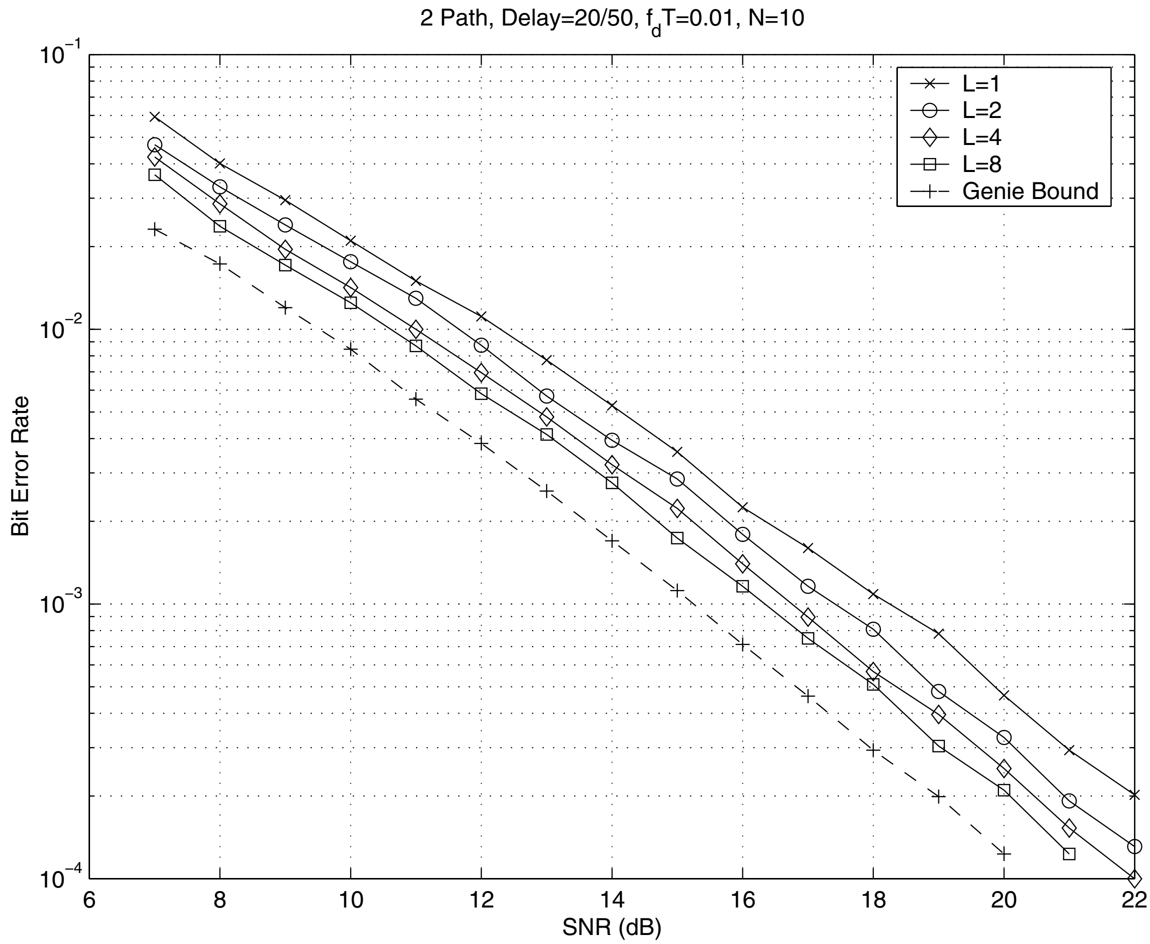

Fig. 23. Bit error rate of EM receiver for various packet lengths ($N = 10$).

Table VII. Summary of results from Fig. 23.

BER	Genie$\rightarrow L = 1$	Genie$\rightarrow L = 2$	Genie$\rightarrow L = 4$	Genie$\rightarrow L = 8$
10^{-2}	2.8 dB	2.1 dB	1.5 dB	1.1 dB
10^{-3}	3.0 dB	2.2 dB	1.5 dB	1.1 dB

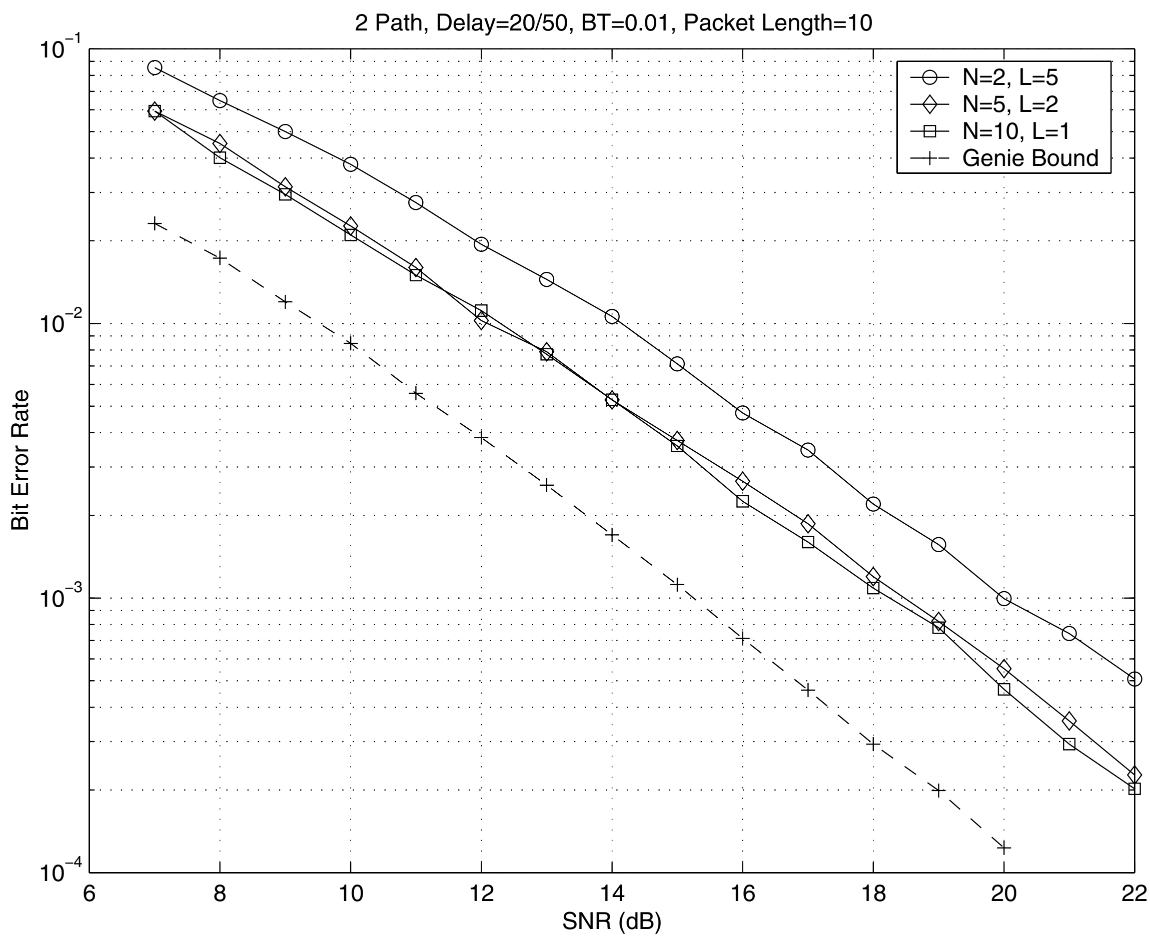

Fig. 24. Bit error rate of EM receiver for various insertion periods (packet length = 10).

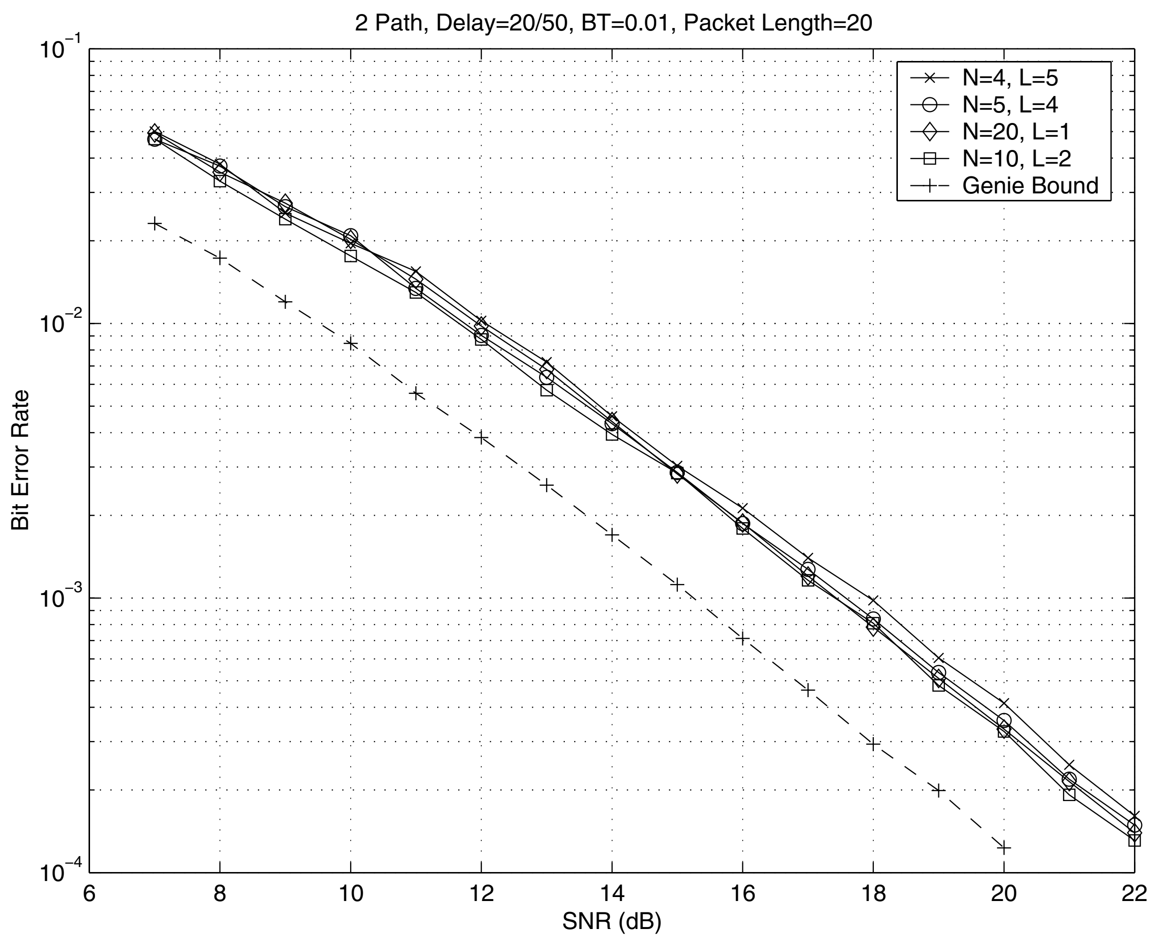

Fig. 25. Bit error rate of EM receiver for various insertion periods (packet length = 20).

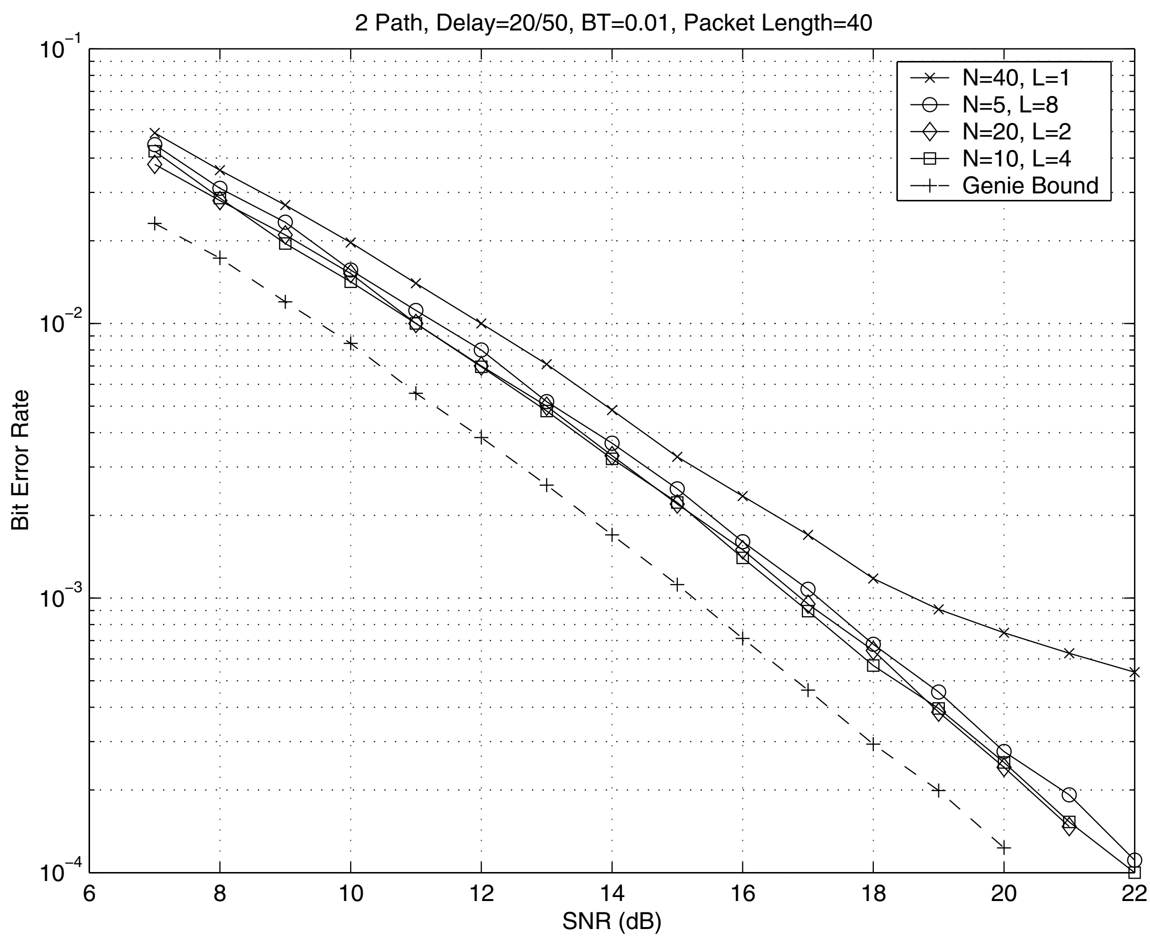

Fig. 26. Bit error rate of EM receiver for various insertion periods (packet length = 40).

2. Mean Square Error of Fading Estimates

For these simulations, the mean square error (MSE) of the final fading estimates from the EM receiver is computed. The first two simulations show the dependence of the MSE on SNR. The last four simulations show the dependence of MSE on the symbol insertion rate.

Figs. 27 through 32 show the simulation results. Fig. 27 and Fig. 28 show that the MSE of the fading estimate degrades as the normalized Doppler shift (f_dT) of the fading increases. Fig. 28 also shows how the MSE of the fading is drastically affected when the symbol insertion rate is below the Nyquist rate established by f_dT. Figs. 29 through 32 show the symbol insertion period that minimizes the MSE of the fading for a given f_dT. These plots also demonstrate the dramatic effect of not satisfying the Nyquist rate. Coupled with the results from the previous subsection, these results can determine the proper symbol insertion rate and packet structure required for a given application.

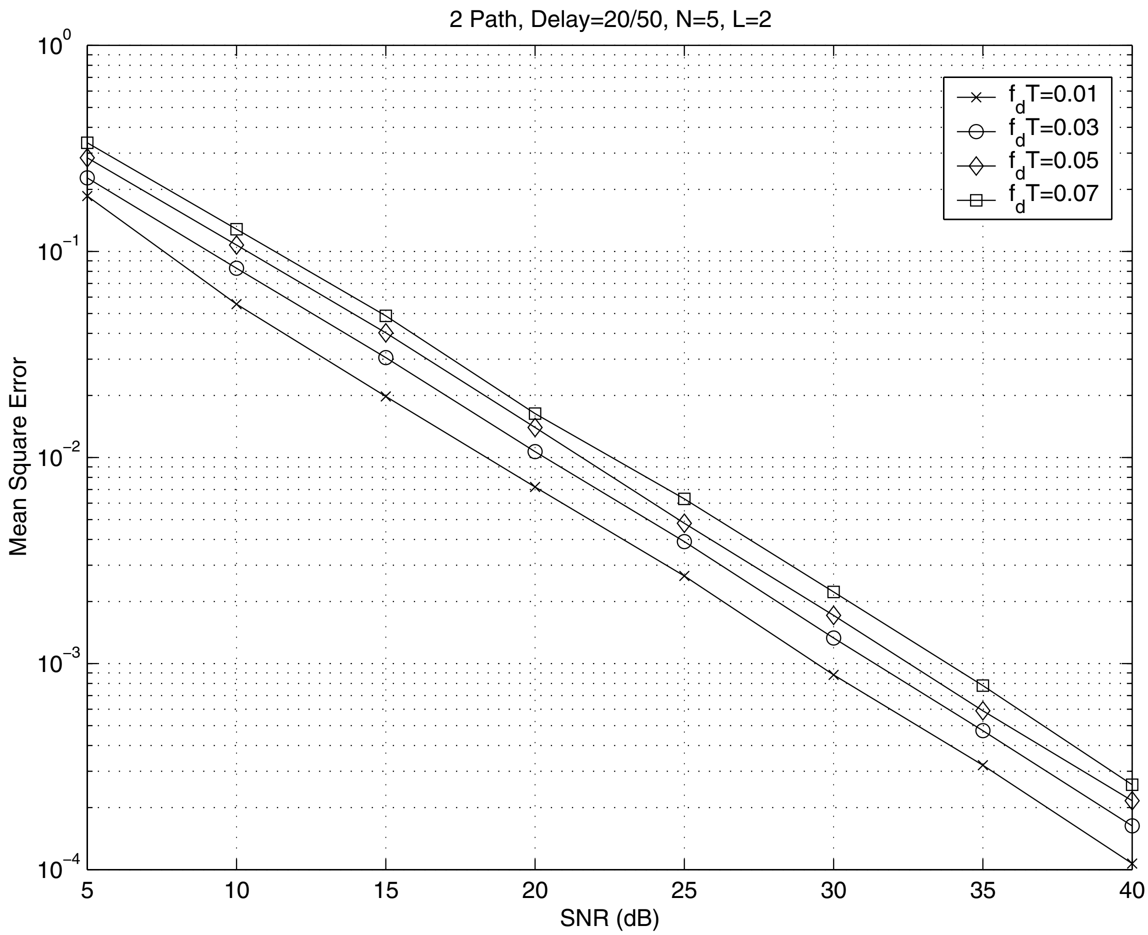

Fig. 27. Mean square error of EM fading estimates ($N = 5$, $L = 2$).

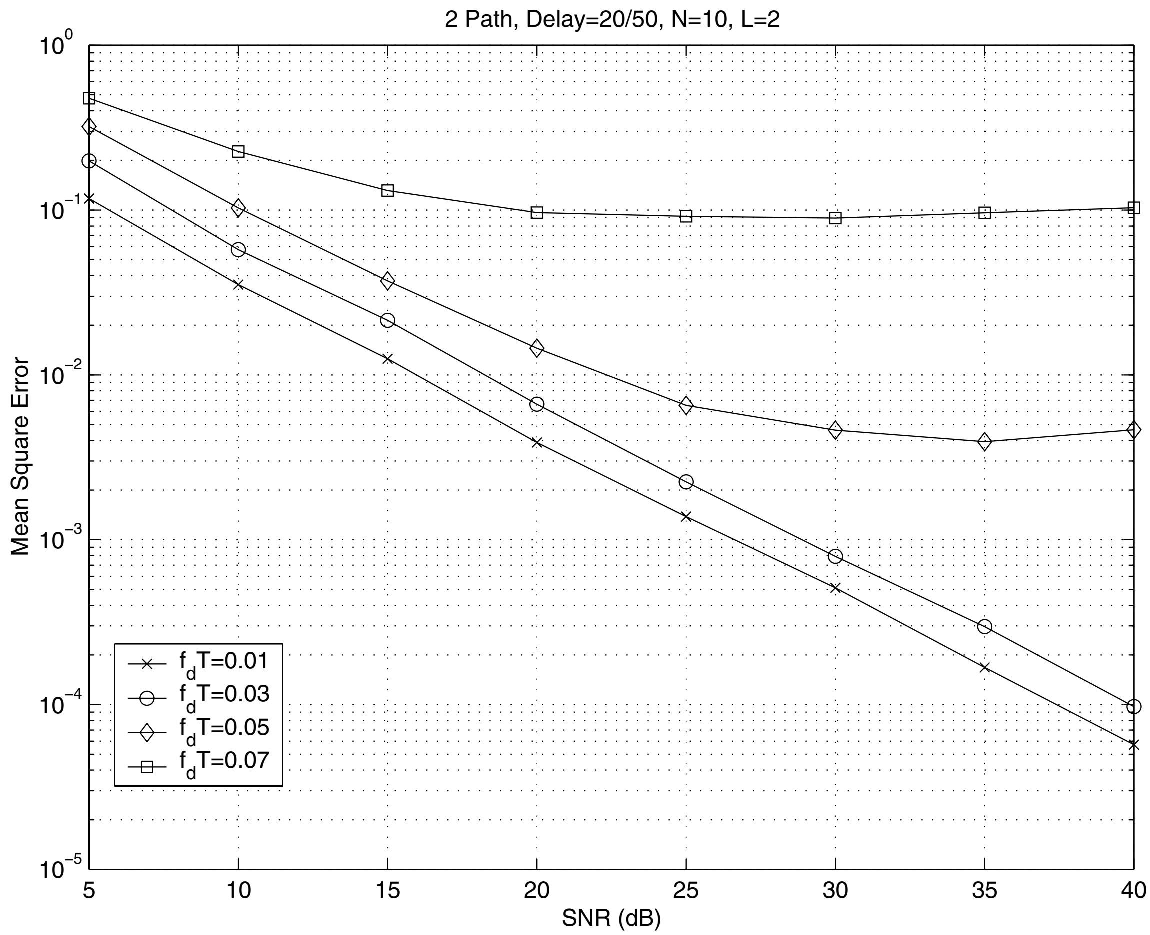

Fig. 28. Mean square error of EM fading estimates ($N = 10$, $L = 2$).

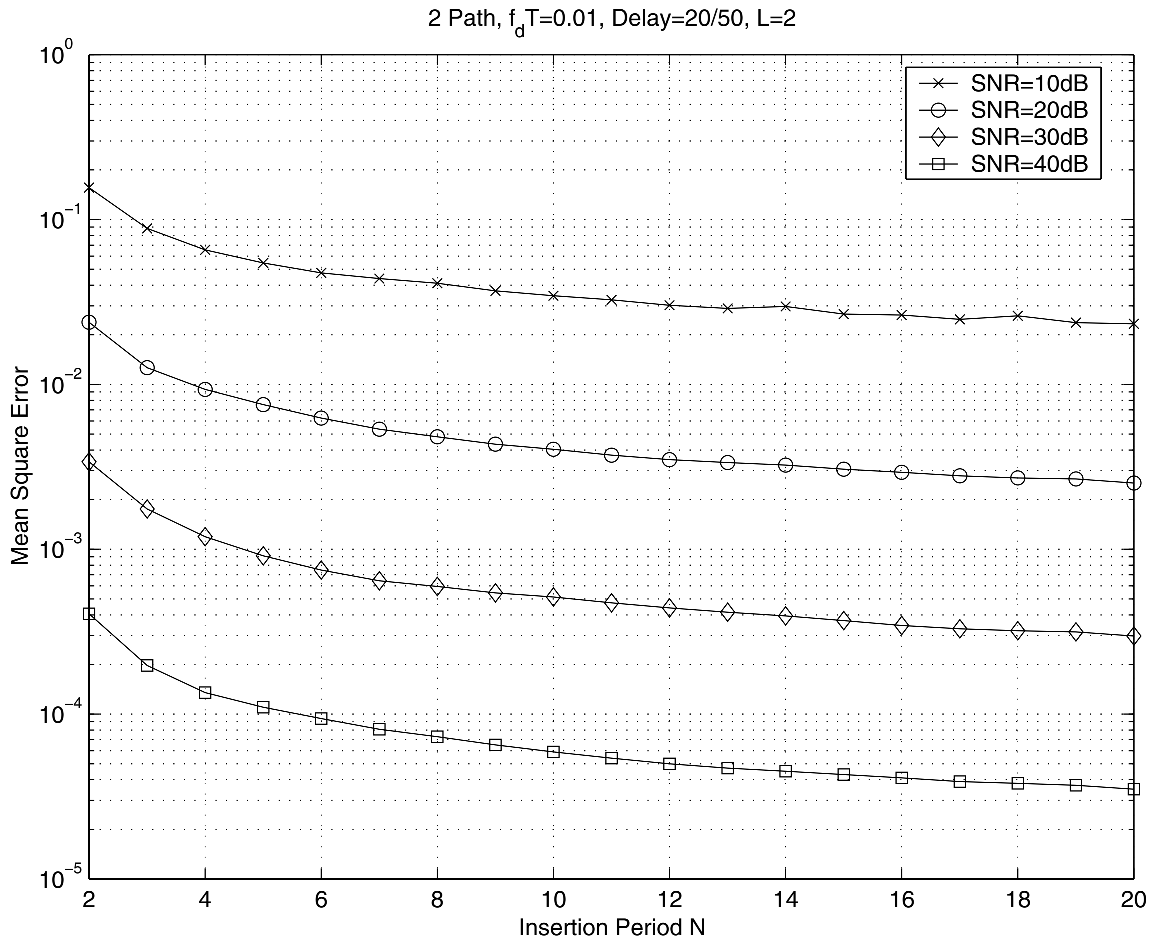

Fig. 29. MSE of EM fading estimates as a function of insertion period N ($f_dT = 0.01$).

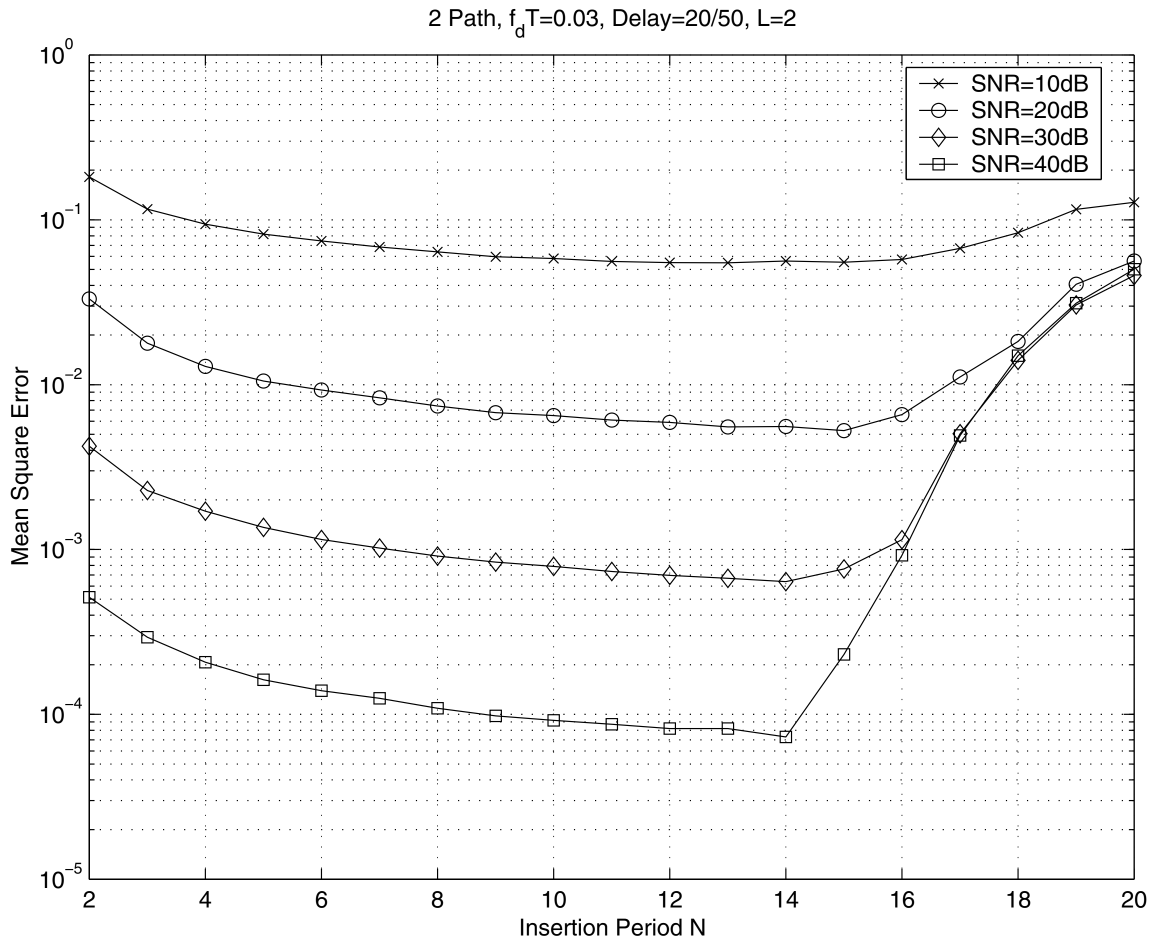

Fig. 30. MSE of EM fading estimates as a function of insertion period N ($f_d T = 0.03$).

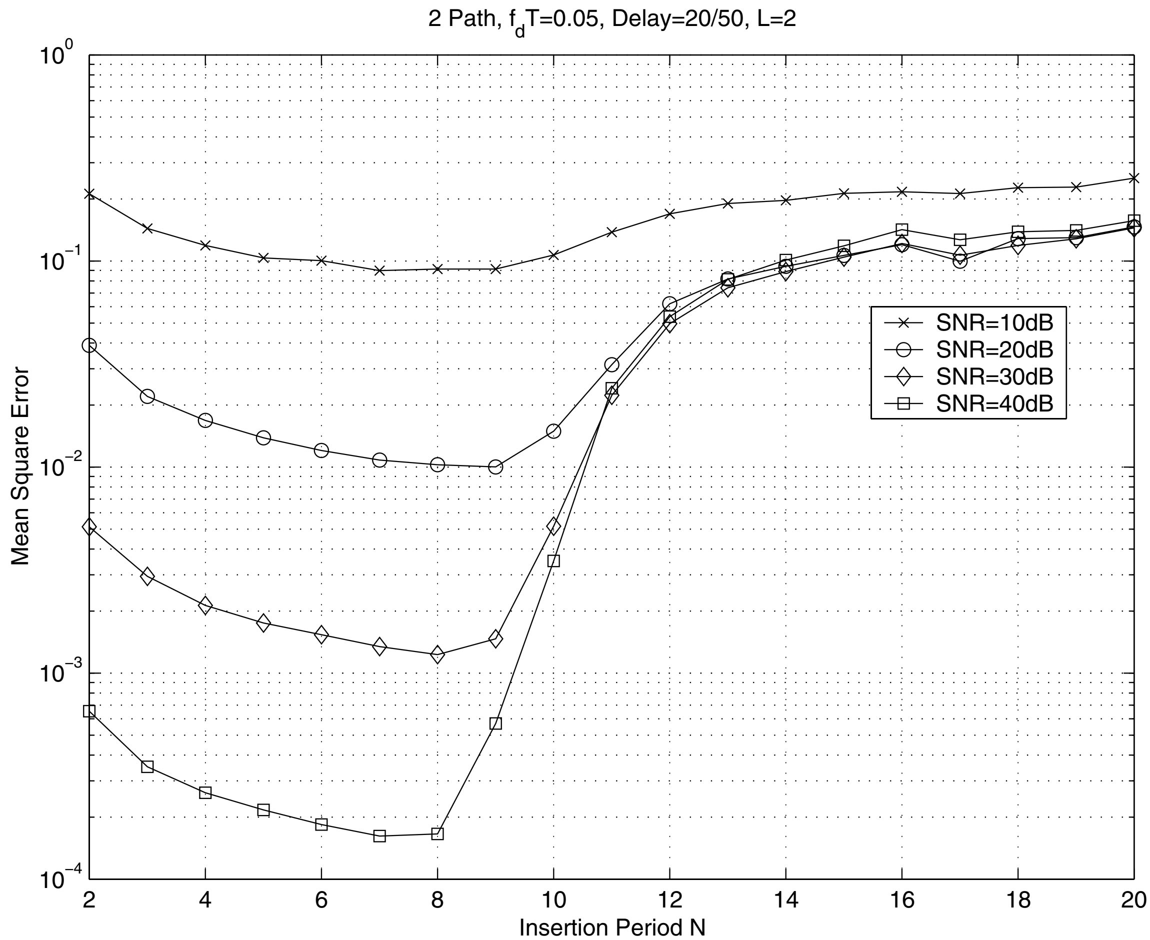

Fig. 31. MSE of EM fading estimates as a function of insertion period N ($f_d T = 0.05$).

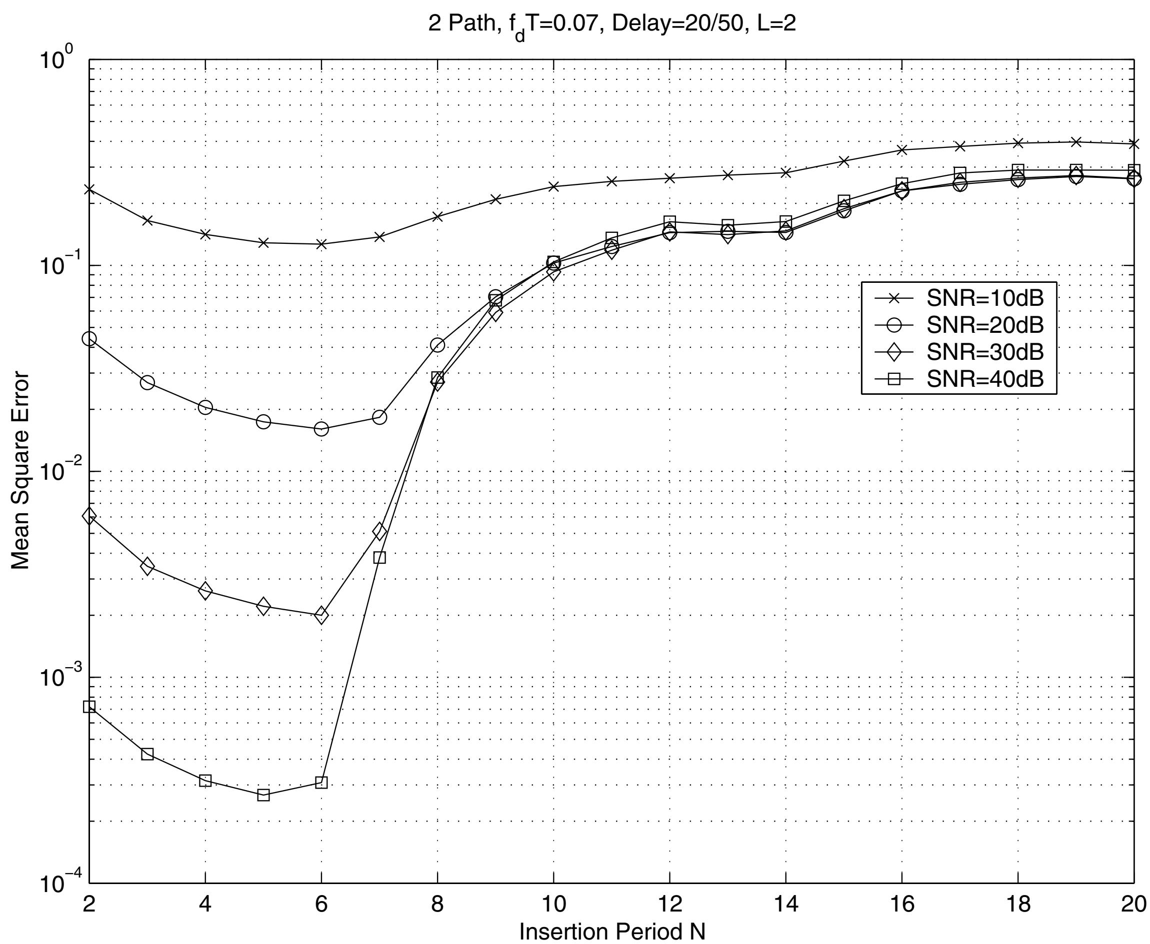

Fig. 32. MSE of EM fading estimates as a function of insertion period N ($f_dT = 0.07$).

CHAPTER VI

CONCLUSION

This thesis investigated receiver design for multipath fading channels. The channel model was chosen to closely resemble a typical land mobile fading channel. The major conclusions of this thesis are:

- The EM receiver for the two path multipath channel was shown to have a similar performance to a two channel diversity system. The summation of independently fading signals at the receiver can be used to provide diversity gain even though the signals are unresolved.

- The EM receiver was shown to have a performance close to the optimum receiver with significantly reduced complexity. At normalized fading Doppler shifts of 0.03 and below, the EM receiver can be within a dB of the optimum receiver.

- The EM receiver was shown to have a superior performance to the best case RAKE receiver. When the signals are unresolved, the RAKE receiver interferes with itself and produces an error floor. The EM receiver is designed to overcome the unresolved multipath.

- The performance of the EM receiver can be improved by carefully selecting the packet structure. Increasing the packet length was shown to improve performance. The reduced complexity allows the EM receiver to process large blocks of data and therefore exceed the performance of an optimum receiver that uses a smaller packet length. The choice of the pilot symbol insertion rate is dependent on the Doppler shift of the fading. It is critical that the insertion rate is chosen to satisfy the Nyquist rate for the worst case Doppler shift encountered.

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APPENDIX A

MATLAB CODE FOR EM BASED RECEIVER SIMULATION

```
% EM_fast.m
% EM Algorithm for Complex Valued Non-Static 2 Path Multipath Channel
% Waveform Channel, 50 samples/bit

% Simulation parameters
  % Output filename
thefilename = 'c:\joe\research\results\EM_fast_waveform.txt';
%thefilename = 'd:\classes\research\final\results.txt';
  % max_errors determines how long the simulation will run
max_errors = 1000;
  % delay_samples must be < 50
delay_samples = 20;
  % Make sure BT*fading_array_length > 3
  % fading_array_length should be a power of 2
BT = 0.01;
fading_array_length = 16384;
N = 5;
L = 2;
packet_length = N*L + 1;
  % SNRdB_array defines which SNR's to simulate
SNRdB_array = [10 12 14 16 18 20 22 24];

% Define some constants
pi = 3.141592654;
i = 1;

% Define square pulse p
p = 1:50;
p = p./p;
Eb = 2*(N/(N-1));

D = eye(packet_length,packet_length);
Q = eye(packet_length);
for n=1:packet_length
   for m=1:packet_length
      Q(n,m) = besselj(0,2*3.141592654*BT*(n-m));
   end
end
S = 0*eye(packet_length);
for k=1:packet_length
   for j=1:packet_length
      if(k == j)
         S(k,j) = (50-delay_samples)/50.0;
```

```
      end
      if(k-j == 1)
         S(k,j) = delay_samples/50.0;
      end
      if(k-j == -1)
         S(k,j) = 0;
      end
      if(abs(k-j) > 1)
         S(k,j) = 0;
      end
   end
end

alpha_prev = rand(2,packet_length);
r = rand(2,packet_length);
alpha = alpha_prev;

ber_array = 0;
est_ber_array = 0;
theoretical = 0;
counter = 1;
total_trials = 0;
start_time = fix(clock)
for SNRdB = SNRdB_array
   fading_array1 = clarke(ceil(fading_array_length*BT),BT,1,1);
   fading_array2 = clarke(ceil(fading_array_length*BT),BT,1,1);
   fading_count1 = 1;
   fading_count2 = 1;
   error_count = 0;
   est_errors = 0;
   trials = 0;
   while error_count < max_errors
      % Create message packet
      bit_array = 2*(rand(1,packet_length)>0.5)-1;
      for i=0:L
         bit_array(i*N+1) = 1;
      end
      % Create transmitted signal
      TXsignal = 1:(packet_length*50);
      for i=1:packet_length
         TXsignal(50*(i-1)+1:50*i) = bit_array(i)*p;
      end

      if fading_count1 > (fading_array_length - packet_length - 10)
         fading_array1 = clarke(ceil(fading_array_length*BT),BT,1,1);
         fading_array2 = clarke(ceil(fading_array_length*BT),BT,1,1);
         fading_count1 = 1;
         fading_count2 = 1;
      end
      % Introduce multipath
      RXsignal = 0*(1:(packet_length*50+delay_samples));
```

```
for i=1:packet_length
   RXsignal(50*(i-1)+1:50*i) = fading_array1(fading_count1)*
                                    TXsignal(50*(i-1)+1:50*i);
   fading_count1 = fading_count1 + 1;
end
for i=1:packet_length
   RXsignal(50*(i-1)+1+delay_samples:50*i+delay_samples) =
       RXsignal(50*(i-1)+1+delay_samples:50*i+delay_samples) +
       fading_array2(fading_count2)*TXsignal(50*(i-1)+1:50*i);
   fading_count2 = fading_count2 + 1;
end

% Introduce AWGN
SNR = 10^(SNRdB/10);
No = 50*Eb/(SNR);
noise = sqrt(No/2)*randn(1,packet_length*50+delay_samples) +
   sqrt(-1)*sqrt(No/2)*randn(1,packet_length*50+delay_samples);
RXsignal = RXsignal + noise;

% Receiver

% Get matched filter samples
for i=1:packet_length
   r(1,i) = RXsignal(50*(i-1)+1:50*i)*p'/50;
   r(2,i) = RXsignal(50*(i-1)+1+delay_samples:50*i+
                                  delay_samples)*p'/50;
end

% Form initial fading estimate
%LMMSE - Begin
D = 0*eye(packet_length);
for i=0:L
   D(i*N+1,i*N+1) = 1;
end
M = eye(packet_length);
for i=0:L
   for j=0:L
      if i~=j
         M(i*N+1,j*N+1) = Q(i*N+1,j*N+1);
      end
   end
end
R = [M+S*M*S'+Eb*eye(packet_length)/(SNR), M*S+S*M+Eb*S/(SNR);
   S'*M+M*S'+Eb*S'/(SNR), S'*M*S+M+Eb*eye(packet_length)/(SNR)];
temp1 = (R^-1)*[r(1,:).'; r(2,:).'];
alpha_prev(1,:) = ([Q*D Q*D*S]*temp1).';
alpha_prev(2,:) = ([Q*D*S' Q*D]*temp1).';
%LMMSE - End

% Compute initial bit estimates - Viterbi Algorithm
   for k=1:packet_length
```

```
   sum1 = 0;
   for m=1:2
      sum1 = sum1 + alpha_prev(m,k)'*r(m,k);
   end
   y(k) = sum1;
end
Node1Metric = 0;
Node2Metric = 0;
Path1 = 0*(1:packet_length);
Path2 = 0*(1:packet_length);
for k = 1:packet_length
   if(k == 1)
      x1 = 0;
   end
   if(k ~= 1)
      x1 = (delay_samples/50.0)*alpha_prev(1,k)'*
                                    alpha_prev(2,k-1);
   end
   %Compute metrics
   M1 = real(y(k) - x1);
   M2 = real(-y(k) + x1);
   M3 = real(y(k) + x1);
   M4 = real(-y(k) - x1);
   if (M1+Node1Metric) > (M3+Node2Metric)
      Node1MetricNext = M1 + Node1Metric;
      Path1Next = Path1;
   else
      Node1MetricNext = M3 + Node2Metric;
      Path1Next = Path2;
   end
   if (M2+Node1Metric) > (M4+Node2Metric)
      Node2MetricNext = M2 + Node1Metric;
      Path2Next = Path1;
   else
      Node2MetricNext = M4 + Node2Metric;
      Path2Next = Path2;
   end
   % Update trellis
   Node1Metric = Node1MetricNext;
   Node2Metric = Node2MetricNext;
   Path1 = Path1Next;
   Path2 = Path2Next;
   Path1(k) = 1;
   Path2(k) = -1;
end
% Make bit decision
if Node1Metric > Node2Metric
   bits_prev = Path1;
else
   bits_prev = Path2;
end
```

```
for i=0:L
   bits_prev(i*N+1) = 1;
end
for k=1:packet_length
   if bits_prev(k) ~= bit_array(k)
      est_errors = est_errors + 1;
   end
end

% EM Algorithm
number = 1;
done = 0;
steps = 0;
while(done == 0)
   % Compute new fading estimates
   for i=1:packet_length
      D(i,i) = bits_prev(i);
   end
   R = [D*Q*D+S*D*Q*D*S'+Eb*eye(packet_length)/(SNR),
      D*Q*D*S+S*D*Q*D+Eb*S/(SNR); S'*D*Q*D+D*Q*D*S'+Eb*S'/(SNR),
      S'*D*Q*D*S+D*Q*D+Eb*eye(packet_length)/(SNR)];
   temp1 = (R^-1)*[r(1,:).'; r(2,:).'];
   alpha(1,:) = ([Q*D Q*D*S]*temp1).';
   alpha(2,:) = ([Q*D*S' Q*D]*temp1).';

   % Compute new bit estimates - Viterbi Algorithm
   for k=1:packet_length
      sum1 = 0;
      for m=1:2
         sum1 = sum1 + alpha(m,k)'*r(m,k);
      end
      y(k) = sum1;
   end
   Node1Metric = 0;
   Node2Metric = 0;
   Path1 = 0*(1:packet_length);
   Path2 = 0*(1:packet_length);
   for k = 1:packet_length
      if(k == 1)
         x1 = 0;
      end
      if(k ~= 1)
         x1 = (delay_samples/50.0)*alpha(1,k)'*alpha(2,k-1);
      end
      %Compute metrics
      M1 = real(y(k) - x1);
      M2 = real(-y(k) + x1);
      M3 = real(y(k) + x1);
      M4 = real(-y(k) - x1);
      if (M1+Node1Metric) > (M3+Node2Metric)
         Node1MetricNext = M1 + Node1Metric;
```

```
            Path1Next = Path1;
         else
            Node1MetricNext = M3 + Node2Metric;
            Path1Next = Path2;
         end
         if (M2+Node1Metric) > (M4+Node2Metric)
            Node2MetricNext = M2 + Node1Metric;
            Path2Next = Path1;
         else
            Node2MetricNext = M4 + Node2Metric;
            Path2Next = Path2;
         end
         % Update trellis
         Node1Metric = Node1MetricNext;
         Node2Metric = Node2MetricNext;
         Path1 = Path1Next;
         Path2 = Path2Next;
         Path1(k) = 1;
         Path2(k) = -1;
      end
      % Make bit decision
      if Node1Metric > Node2Metric
         bits = Path1;
      else
         bits = Path2;
      end

      % Setup the next iteration
      if(bits_prev == bits)
         done = 1;
      end
      bits_prev = bits;
      steps = steps + 1;
      if(steps > 50)
         done = 1;
      end
   end
   number_wrong = 0;
   for i=0:L
      if bits(i*N+1) == -1
         number_wrong = number_wrong + 1;
      end
   end
   if number_wrong == L+1
      bits = -bits;
   end
   for i=0:L
      bits(i*N+1) = 1;
   end
   for k = 1:packet_length
      if bits(k) ~= bit_array(k)
```

```
            error_count = error_count + 1;
         end
      end
      trials = trials + packet_length - (L+1);
   end
   SNRdB
   fix(clock)
   trials
   total_trials = total_trials + trials;
   ber = error_count/trials
   ber_array(counter) = ber;
   est_ber = est_errors/trials
   est_ber_array(counter) = est_ber;
   counter = counter + 1;
end
end_time = fix(clock);
total_trials
total_time = etime(end_time,start_time)
ave_trials_per_sec = total_trials/total_time

%Plot simulation results
semilogy(SNRdB_array,ber_array,SNRdB_array,est_ber_array)
xlabel('SNR (dB)')
ylabel('BER')
title('Bit Error Rate vs SNR')
legend('EM Alg','Init Est',0)

%Write data to a file
thefile = fopen(thefilename,'w');
fprintf(thefile,'File based on EM_fast_waveform.m\r\n');
fprintf(thefile,'2 Path\r\n');
fprintf(thefile,'max_errors = %d\r\n',max_errors);
fprintf(thefile,'Delay = %d/50\r\n',delay_samples);
fprintf(thefile,'BT = %f\r\n',BT);
fprintf(thefile,'N = %d\r\n',N);
fprintf(thefile,'L = %d\r\n',L);
fprintf(thefile,'packet_length = %d\r\n\',packet_length);
fprintf(thefile,'\r\nTotal trials   = %d\r\n',total_trials);
fprintf(thefile,'Total sim time = %d sec\r\n', total_time);
fprintf(thefile,'Ave trials/sec = %f\r\n',ave_trials_per_sec);
fprintf(thefile,'\r\nSNR, EM Alg, Init Est\r\n');
for j=1:length(SNRdB_array)
   fprintf(thefile,'%d, %f, %f\r\n',SNRdB_array(j),
                   ber_array(j),est_ber_array(j));
end
fclose(thefile);


%----------------------------------------------
%  File: clarke.m
%----------------------------------------------
```

```
%
%       Fading channel simulator based on Clarke's model:
%
%       Input parameters :
%               mini : # of points to represent Doppler spread
%               fD   : Doppler frequency (Hertz) = v*Fc/c
%               Br   : Bit rate (bits per sec.)
%               cpb  : # of coefficients per bit
%                       note : cpb may > 1 for fast fading,
%                                      < 1 for slow fading.
%
%       Output : Vector of complex Rayleigh fading coefficients
%
function y=clarke(mini,fD,Br,cpb)
%
dF=fD/(mini-1);
%       maximum frequency resolution
%
maxi=Br*cpb/dF;
%       maximum total points needed
%
maxi=2^nextpow2(maxi);
%       take bigger power of 2 for IFFT
%
dF=Br*cpb/(maxi-1);
%       new frequency resolution
%
mini=ceil(fD/dF);
%       new # of points for Doppler
%
pts=0:mini-1;
temp = 0;
for j=-(mini-1):mini-1
   temp = temp + (fD^2-(j*dF)^2)^(-1/2);
end
a = 1/temp;
Dop=(1/sqrt(4))*sqrt(a./sqrt(fD^2-(dF*pts).^2));
%       positive half Doppler spectrum
%
inphase=Dop.*(randn(1,mini)+sqrt(-1)*randn(1,mini));
quadrat=Dop.*(randn(1,mini)+sqrt(-1)*randn(1,mini));
%       inphase & quadrature half
%
inphase(maxi:-1:maxi-mini+2)=conj(inphase(2:mini));
quadrat(maxi:-1:maxi-mini+2)=conj(quadrat(2:mini));
%       copy of conjugate half
%
y=fft(inphase)+sqrt(-1)*fft(quadrat);
%       coefficients in time domain
```

VITA

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